# Liquidity Constraints and Demand for Maturity: The Case of Mortgages* 

Alessandro Ferrari ${ }^{\dagger} \quad$ Marco Loseto ${ }^{\ddagger}$

November 2023


#### Abstract

Using administrative data on mortgages issued in Italy between 2018 and 2019, this paper estimates loan demand elasticities to maturity and interest rate. We find that households are responsive to both contract terms: a $1 \%$ decrease in interest rate increases the average loan size by $0.22 \%$ whereas a commensurable increase in maturity increases loan demand by $0.30 \%$. Things change substantially when moving along the distribution of contract maturities: short term borrowers are unresponsive to their contract length while maturity elasticities are higher for long term borrowers. Through the lens of a life-cycle model with long-term installment debt we show how to use the estimated maturity elasticity to test for the presence of credit constraints.


Keywords: mortgage, household finance, credit demand, maturity.

JEL Codes: D12, D14, D15, G11, G51.

[^0]
## 1 Introduction

Households rely on credit markets to purchase goods that would not be affordable to them otherwise. Classic examples are durable goods like housing and cars which are typically financed via mortgages and auto loans, respectively. These latter are complex financial contracts where price and non-price dimensions interact together in determining borrowers' total cost of credit.

In this paper we join a recent literature that studies how credit demand responds to both price and non-price dimensions. Quantifying borrowers' elasticities to non-price dimensions such as loan maturity is important for at least two reasons. First, non-price elasticities inform regulators about how lenders can exercise market power by leveraging contract terms other than price. Second, knowledge of non-price elasticities might help policy makers both in orienting new interventions toward more responsive margins and/or in evaluating measures already in place.

Specifically, we use administrative data on mortgages issued in Italy between 2018 and 2019 and start by documenting two important empirical facts. First, as Figure 1 shows, even for loan of the same size, there is substantial heterogeneity in the duration of realized mortgages ${ }_{\square}^{1}$ The distribution of maturities is skewed toward longer terms, as expected, since maturities below 20 years typically require substantial periodic payments, which might be accessible only to wealthy/high income borrowers. Second, in Figure 3, we highlight a strong and positive correlation between the amount and maturity of the loan which indicates the possibility of credit demand being significantly upward sloping in the offered maturity. If this were the case, when offered a longer maturity borrowers would increase their debt balance and presumably increase current consumption, consistent with the presence of binding liquidity constraints. $\int^{2}$

To test this intuition, we exploit the observed variation in contract maturities to estimate the loan demand elasticity to maturity separately from the loan demand elasticity to interest rates. The basic identification challenge is that realized contract terms and loan demand are jointly endogenous. To isolate exogenous variation in the realized interest rates and maturities we complement our administrative dataset with data on mortgage offers posted by lenders on the main Italian mortgage online platform MutuiOnline.it $]^{3}$ These offers are collected by submitting fictitious applications (varying households' characteristics) to the online platform

[^1]and then recording the mortgage terms each lender is willing to offer to each possible combination of characteristics. Under the institutional agreement in which lenders and the online platform operate, the online offers represent lenders' pre-approval decisions and, conditional on the accuracy of the information provided by the applicant, the offered terms are binding. This implies that posted offers are realistic and that the information an applicant is required to submit through the platform is enough for lenders to plausibly price a mortgage.

For each realized contract in our administrative dataset we match an offered contract from our data on online offers. This allows us to instrument the realized interest rate and maturity with the corresponding terms of the matched offered contract. The main identifying assumption is that variation in online offers reflects changes in lenders' supply policies that are excluded from loan demand. Overall, we find that the average borrower response to an exogenous change in maturity is slightly stronger than its response to an exogenous change in interest rate. Holding everything else constant, a $1 \%$ increase in contract maturity increases credit demand by $0.30 \%$ whereas $1 \%$ decrease in interest rate increases demand by $0.22 \%$.

We further show that this high sensitivity to maturity is entirely driven by long-term borrowers. Our estimates suggests that short-term borrowers (i.e., contracts with maturities below 15 years) are unresponsive to changes in maturity while for long-term borrowers (i.e., contracts with maturities above 15 years) the maturity elasticity is larger than for the average borrower.

To interpret our estimates, Section 3 develops a simple life-cycle model with long-term installment debt. At $t=0$, to increase current consumption households have access to a noncallable long-term debt contract with given rate and customizable duration whose principal and interests are repaid periodically in equal installments over the life of the contract. The maximum loan size is bounded by a debt-to-income constraint ensuring that periodic payments can be covered by labor income only, whereas the contract length is bounded by the maximum available maturity.

In our simple model loan demand for constrained borrowers slopes upward in the maximum available maturity: longer contracts allow constrained borrowers to increase the size of their loan and in turn increase current consumption while maintaining the same debt-to-income ratio. On the other hand, unconstrained households that issue a positive amount of debt are unresponsive to their contract duration. Thus, our model suggests that maturity elasticities can be used to test for the presence of liquidity constraints. ${ }_{4}^{4}$

Altogether, the contribution of this paper is twofold. First, we quantify the maturity elasticity of loan demand for the case of mortgages which, to the best of our knowledge, is still

[^2]unexplored ${ }_{5}^{5}$ Second, we exploit maturity elasticities to assess the importance of credit constraints in mortgage markets. Our baseline estimates suggest that binding credit constraints might be important in our setting. This evidence is further confirmed when we look at maturity elasticities separately for short and long term borrowers. We expect the former to be on average wealthier and less credit constrained, as short term loans generally require higher payments. Our estimates corroborate this intuition- short term borrowers are unresponsive to changes in their contract duration while long term borrowers are more responsive to changes in maturity than the average borrower.

Turning to policy, our findings suggest that interventions targeting the supply of maturity could have a significant impact on credit demand. Furthermore, according to our estimates, these interventions will mostly affect credit constrained borrowers without distorting credit demand too much for unconstrained borrowers. Thus, compared to interest rate policies, maturity policies might be better suited for interventions that aim at affecting constrained borrowers and stimulating demand through credit accessibility.

The rest of the paper proceeds as follows. Section 2 reviews the literature, Section 3 describes the theoretical framework, Section 4 describes the data, Section 5 discusses identification and presents our results, Section 6 describes the policy implications and, finally, Section 7 concludes.

## 2 Related Literature

Using survey data on an hypothetical automobile financing decision Juster and Shay (1964) were the first to study borrowers' sensitivity to both interest rate and maturity. Their evidence highlights two important empirical facts that are consistent with our empirical findings. First, the extent with which constrained borrowers respond to changes in finance charges (i.e., interest rate) is entirely driven by the implied changes in payments size. Second, holding finance rates constant, debt positions of unconstrained borrowers will be unaffected by lengthening of the maximum available maturity as opposed to constrained borrowers which will effectively borrow more.

[^3]With the increasing availability of granular microdata on household debt decisions more recent contributions pushed this literature substantially further. Attanasio et al. (2008) investigate the relevance of borrowing constraints using data from the Consumer Expenditure Survey on auto loans contracts. They find that maturity elasticities are on average higher than interest rate elasticities except for high income households and interpret this as evidence of the presence of binding credit constraints in the auto loan markets. Similarly, Karlan and Zinman (2008) test borrowers' sensitivity to interest rates and maturity in less-developed countries exploiting a randomized control trial conducted in South Africa. They also find significant and positive maturity elasticities consistent with borrowers being credit constrained and thus more concerned about monthly cash flows than total finance charges.

In a more recent contribution, Argyle et al. (2020) use discontinuities at different FICO scores in lenders pricing and maturity rules to identify car-loan demand elasticities to both interest rate and maturity. Their results suggest that consumers worry about managing payment sizes when making debt decisions- on average car loan demand is significantly sensitive to maturity while being quite unresponsive to interest rate charges. They argue that these findings are consistent not only with the presence of liquidity constraints but also with the presence of behavioral frictions such as segregated mental accounts and monthly payment bunching at salient amounts. Grunewald et al. (2020) focus instead on the supply side of car-loans and find that auto dealers have an incentive to increase loan charges (i.e., interest rates) instead of car prices because consumers are less elastic to changes in their loan interest rate than to changes in car prices. According to the paper, several explanations might drive this asymmetry in consumers' responsiveness including credit constraints, which make borrowers less sensitive to finance charges and, more generally, consumers inability to correctly assess loan prices.

Our paper contributes to this literature by providing, to the best of our knowledge, the first estimates of maturity elasticities of loan demand for the mortgage market which is, for most countries, arguably the biggest market in which households debt resides. We find that similarly to auto-loans, mortgage demand slopes upward in contract maturities but, differently from auto-loans, mortgage demand is also substantially responsive to interest rate. This is intuitive because mortgages are typically longer than auto-loans which means that monthly payments will be also quite responsive to changes in interest charges and, therefore, borrowers will be responsive to interest rates even when liquidity constrained.

More broadly we join a relatively small but growing literature that studies the role of household debt maturity in credit markets. Argyle et al. (2021) find that shocks to the supply of maturity impact durable goods prices; they show that car prices adjust in response to exogenous variation in auto-loan maturities and attribute it to borrowers' ability to negotiate lower
prices. Ganong and Noel (2020) show that policy interventions aimed at improving borrowers' liquidity such as maturity extension programs have large effects on borrowers' default and consumption decisions even when these extensions are not accompanied by a principal reduction. For the case of unsecured personal loans, Hertzberg et al. (2018) study the role of maturity in screening borrowers. They exploit a natural experiment and show that borrowers self-select into longer maturity when their ability to repay the loan is lower. Finally, in a recent review paper, Zinman (2015) points out that measuring credit constraints still remains a challenge and estimating maturity elasticities is an informative yet overlooked approach to do so.

## 3 Conceptual Framework

In this section we consider a simple life-cycle model with long-term household debt. Even though mortgage debt is our empirical application, we do not model housing and mortgage choices explicitly because in practice Italian mortgages resemble quite closely non-callable long-term loans. In Italy prepayment and refinance rates are low and most mortgages are held up to maturity ${ }^{6}$ Similarly, default rates are also extremely low because Italian banks typically hold mortgages on their balance sheets and thus apply tough screening policies. ${ }^{7}$ As a consequence, we allow households to issue a simple non-callable long-term debt to finance current consumption and abstract away from modelling prepayment and default options. Moreover, we do not model housing as an asset because it is uncommon for Italian households to open home equity lines of credit or, in general, to issue debt against their home equity. ${ }^{8}$

In what follows, Subsection 3.1 outlines the model and presents the optimality conditions while Subsection 3.2 compares loan demand of constrained and unconstrained borrowers.

### 3.1 Model

A finitely lived hand-to-mouth household $(\mathrm{HH})$ is endowed with initial wealth $a_{0}$ and maximizes its discounted present utility by making a one-shot long-term debt decision ${ }^{9}$ At $t=0$, to finance current consumption, HH can issue a non-callable, fully amortizing long-term debt with fixed rate $r$ and maximum duration $\tau$. In each period HH also obtains stochastic labor

[^4]income $y_{t} .{ }^{10}$
In sequence form HH's maximization problem reads:
\[

$$
\begin{align*}
V(r) \equiv & \max _{b_{0}, s} u\left(w_{0}+b_{0}\right)+\mathbb{E}_{0}\left[\int_{0}^{s} e^{-\rho t} u\left(y_{t}-m\left(b_{0}, r, s\right)\right) d t+\int_{s}^{T} e^{-\rho t} u\left(y_{t}\right) d t\right]  \tag{1}\\
\text { s.t. } & b_{0} \leq \frac{\phi y_{0}}{m(r, s)}  \tag{2}\\
& b_{0} \geq 0  \tag{3}\\
& s \leq \tau \tag{4}
\end{align*}
$$
\]

where $w_{0}=a_{0}+y_{0}$ denotes the initial wealth, constraint (2) is the debt-to-income constraint, (3) is a saving constraint and (4) bounds the debt maturity $s$ to a maximum of $\tau \sqrt{11}$ We assume that the debt installments are equally sized so that the periodic payment function is linear in $b_{0}$ i.e., $m\left(b_{0}, r, \tau\right)=b_{0} m(r, \tau){ }^{12}$

Letting $\mu, \eta$ and $\gamma$ be the Lagrange multipliers on the debt-to-income, saving and maturity constraints respectively, the following optimality conditions must hold;

$$
\begin{align*}
& b_{0}: u^{\prime}\left(w_{0}+b_{0}\right)-\mathbb{E}_{0}\left[\int_{0}^{s} e^{-\rho t} u^{\prime}\left(y_{t}-b_{0} m(r, s)\right) m(r, s) d t\right]+\eta-\mu m(r, s)=0  \tag{5}\\
& s:-\mathbb{E}_{0}\left[\int_{0}^{s} e^{-\rho t} u^{\prime}\left(y_{t}-b_{0} m(r, s)\right) b_{0} m^{\prime}(r, s) d t\right] \\
&+\mathbb{E}_{0}\left[e^{-\rho s} u\left(y_{s}-b_{0} m(r, s)\right)-e^{-\rho s} u\left(y_{s}\right)\right]-\mu b_{0} m^{\prime}(r, s)-\gamma=0  \tag{6}\\
& \mu \geq 0, \quad \mu\left(\phi y_{0}-b_{0} m(r, s)\right)=0  \tag{7}\\
& \eta \geq 0, \quad \eta b_{0}=0  \tag{8}\\
& \gamma \geq 0, \quad \gamma(\tau-s)=0 \tag{9}
\end{align*}
$$

Equation (5) is key to our analysis as it pins down the loan demand $b_{0}$ as a function of both interest rate $r$ and contract maturity $s$. The optimal amount of long-term debt trades off the marginal benefit of increasing current consumption with the expected marginal-utility weighted flow of future debt payments. Equation (6) determines instead the demand of maturity as a function of both interest $r$ rate and current debt level $b$. The debt maturity choice trades off the benefits of lower payments over the loan life (i.e., $m^{\prime}(r, s)<0$ ) with the cost of paying additional interest for an extra period ${ }^{13}$ Finally, equations $(7)$ to 9 are the usual complementary slackness conditions.

[^5]
### 3.2 Loan Demand: Constrained vs. Unconstrained

We now turn our attention to households that access credit markets i.e., the ones for whom $b_{0}^{*}>0$ or equivalently $\eta=0$, and compare loan demand of constrained and unconstrained borrowers.

We define liquidity constrained borrowers as the ones that would like to issue more debt to finance current consumption but are unable to do so (i.e., the ones for whom condition (5) is violated and $\mu>0$ ). To borrow more, most of these borrowers will also find optimal to increase their debt maturity as much as possible to relax the debt to income constraint in (2), so that $s=\tau$ and $\gamma>0{ }^{14}$ Their loan demand, as a function of the interest rate $r$ and maximal maturity $\tau$, is then directly pinned down by the binding debt-to-income constraint in equation (2):

$$
\begin{equation*}
b_{0}^{*}(r, \tau)=\frac{\phi y_{0}}{m(r, \tau)} . \tag{10}
\end{equation*}
$$

In response to marginal changes in duration and/or interest rate, liquidity constrained borrowers will adjust their loan size to keep their debt-to-income ratio constant at $\phi{ }^{15}$

Turning to unconstrained borrowers for whom it is optimal to issue some long-term debt (i.e., the ones with $\eta=0$ and $\mu=0$ ), their optimal loan size equalizes the benefit of increasing consumption at $t=0$, measured by the current marginal utility, with the cost of higher loan payments measured by the marginal-utility-weighted discounted flow of future payments as captured by equation (5). Among the unconstrained, the ones that opt for an interior maturity (e.g., $s<\tau$ and $\gamma=0$ ) will be unresponsive to marginal changes in the maximal available maturity $\tau$. On the other hand, it can happen that, even if unconstrained, some borrowers prefer to extend their maturity as much as possible $s=\tau$. In this case, their loan demand will be implicitly defined by:

$$
\begin{equation*}
u^{\prime}\left(w_{0}+b_{0}^{*}\right)=\mathbb{E}_{0}\left[\int_{0}^{\tau} e^{-\rho t} u^{\prime}\left(y_{t}-b_{0}^{*} m(r, \tau)\right) m(r, \tau) d t\right] \tag{11}
\end{equation*}
$$

and will depend on the maximum available maturity $\tau$.
What can we learn about loan demand of constrained and unconstrained borrowers from the previous analysis? The answer is straightforward for constrained borrowers: equation (10) tells us that loan size decreases in $r$ and increases with $\tau$. This is because interest rate and maturity affect loan demand only through the periodic payments $m(r, \tau)$ - holding everything else constant, a contract with higher $r$ (longer $\tau$ ) requires higher (lower) payments.

[^6]For unconstrained borrowers things are a bit different. While an increase in $r$ clearly decreases loan demand $b_{0}^{*}$ because of income and (inter-temporal) substitution effects, the amount of debt issued is likely to be unresponsive to changes in the maximum available duration $\tau$. This is unambiguously true for unconstrained borrowers that do not take advantage of the maximum offered maturity $\tau$ and instead select an interior duration $s<\tau$. For borrowers that instead, even if unconstrained, prefer to maximize their debt maturity (i.e., $s=\tau$ ), the demand response to changes in $\tau$ is ambiguous. Holding everything else constant, a marginal increase in $\tau$ reduces loan payments and allows HHs to consume more in the periods when the loan is repaid (i.e., the RHS of (11) goes down). On the other hand, total borrowing costs will go up because an additional payment is needed to repay the loan i.e., an additional positive term $\mathbb{E}_{0}\left[e^{-\rho \tau} u^{\prime}\left(y_{\tau}-b m(r, \tau)\right) m(r, \tau)\right]$ will be added to the RHS of equation 11). The first effect pushes HHs to borrow more to increase current consumption while the second one pushes them to borrow less.

While theoretically plausible, this last group of borrowers (i.e., the ones that even if unconstrained $(\mu=0)$ choose to maximize debt duration $(\gamma>0)$ ) is likely to be marginal in practice. This is because debt size and duration have a complementary role: both can be used to smooth consumption by transferring resources from tomorrow to today. Therefore, borrowers that would like to extend their contract maturity above $\tau$ should also be willing to borrow as much as possible today (e.g., $\gamma>0$ is likely to also imply that $\mu>0$ ). Overall, we expect both the debt-to-income constraint and the maximum maturity constraint either to bind ( $\mu>0$ and $\gamma>0$ ) or not to bind ( $\mu=0$ and $\gamma=0$ ), whereas the other two possibilities are mostly theoretical artifacts.

To sum up, although highly stylized, this framework provides us with a clear and testable prediction: credit demand of constrained borrowers slopes upward in contract maturity whereas it is flat for unconstrained borrowers. As a consequence, the maturity elasticity of credit demand can be used to assess the importance of liquidity constraints in credit markets- a positive and significant maturity elasticity estimate is consistent with the average borrower being liquidity constrained.

## 4 Data

This section describes the two datasets we employ throughout our analysis. Subsection 4.1 focuses on our first data source which contains administrative data on mortgage loans issued in Italy since 2004, subsection 4.2 presents our dataset on online mortgage offers and subsection 4.3 details some summary statistics of both datasets.

### 4.1 Realized mortgage contracts

Our dataset on realized mortgage contracts combines two different administrative data sources both maintained by the Bank of Italy: the Italian Credit Register (CR) and the Analytical Survey of Lending Rates (TAXIA).

Banking laws require both Italian and foreign lenders operating in Italy to report to CR information about all outstanding loan exposures above 30,000 euros. This includes information on lender and borrower identifiers, type of loan, outstanding debt, guarantees and delinquencies at monthly frequency.

Data on realized loan prices are instead available in TAXIA. This dataset is a random subset of the CR universe and contains quarterly information on both newly issued loans (i.e., flows) and outstanding loans (i.e, stocks). For each newly issued loan we observe detailed contract characteristics including the annual percentage rate (APR), loan size, type of rate (FRM vs. ARM); and some borrower characteristics such as the province of residence, age, gender and nationality. For each outstanding loan we observe the euro amount of interest repaid in each quarter. This latter information is essential as it allows us to recover the maturity of each contract which is central to our analysis yet not directly available in our data. $\sqrt{16}^{16}$

Overall, the lender and borrower identifiers allow us to match and merge the CR data with the TAXIA data and to get comprehensive loan-level information on a random sample of residential Italian mortgage contracts.

### 4.2 Online mortgage offers

We complement our data on realized mortgage contracts with data on online mortgage offers available from the main online mortgage platform in Italy (MutuiOnline.it). The data are collected by submitting monthly (March 2018 - August 2019) fictitious online applications (varying potential borrowers' characteristics) and contain the APR (i.e., interest gross of fees) and the net interest rate each lender is willing to offer to a given potential borrower profile. ${ }^{17}$

A potential borrower profile is a given by a combination of the characteristics and the mortgage terms that a platform user has to specify in order to obtain mortgage offers. The dataset records the rates offered by lenders to 256 potential borrower profiles that are constructed as follows. Each borrower profile is defined by a combination of the following characteristics: age ( 30 or 40 years old), net monthly income ( 2,000 or 4,000 euros), employment type (fixedterm or permanent contract), loan-to-value (LTV of $50 \%, 60 \%, 80 \%$ or $85 \%$ ), maturity ( 10,15 ,

[^7]20 or 30 years) and type of rate (FRM or ARM). Each profile is kept constant both over time and across geographic markets (i.e., the 110 italian provinces) so that within each potential borrower profile we have variation in posted interest rates across three important dimensions: lenders, time and markets.

### 4.3 Descriptive statistics

Table 1 summarizes our administrative data on realized mortgage contracts. We focus on loans issued between March 2018 and August 2019 as this is the period of time for which we also have information on online mortgage offers from the MutuiOnline.it platform. After combining administrative information on monthly balances available in CR with quarterly information on prices and interest payments available in TAXIA, we recovered contract maturities for 186,000 loans. The average mortgage APR is $2.3 \%$ while the average loan size is around 137,000 euros. The latter is slightly higher than the median loan amount of 120,000 euros suggesting that the distribution is left-skewed.

Turning to contract maturities, there is plenty of variation in contract lengths both in the whole sample and when considering contracts with more or less the same loan size (e.g., Figure 11. Most of maturities are in multiple of 5 years but less common maturities are also possible. As shown in Table 1. for the whole sample the average maturity is around 23 years and $75 \%$ of contracts are medium-long term with duration above 20 years. Finally, it is worth pointing out that $70 \%$ of the contracts are fixed rate mortgages. This is reasonable as interest rates were particularly low in this period of time (March 2018 - August 2019) and for most borrowers must have been convenient to lock in a low rate through a fixed contract.

Table 2 presents some summary statistics on online mortgage offers available on the $M u$ tuiOnline.it platform in between March 2018 and August 2019. The table shows means and standard deviations (across lenders, time and markets) of the posted APR, net interest rate, and monthly payment by loan-to-value and maturity ${ }^{18}$ Here three remarks are in order. First, APR increases in LTV consistent with risk-based pricing. 19 Second, longer contracts come with a higher interest rate. This is especially true for FRM (top panel in Table 2) and consistent with lenders requiring a premium for insuring borrowers against interest rate risk. Finally, compared to realized rates, online offered rates seem to be slightly lower and less dispersed ${ }^{20}$ This

[^8]is consistent with online mortgage markets begin more competitive as borrowers' search costs are minimized (i.e., through the platform online borrowers can immediately compare all the available offers).

## 5 Estimation

In this section we turn the spotlight on identification and estimation. We begin by describing the estimation framework and the identification challenges. After that, we outline our identification strategy (Subsection 5.1) and presents our empirical findings (Subsections 5.2 and 5.3).

The main goal is to estimate loan demand elasticities with respect to interest rate and maturity. Letting $Q, R$ and $T$ being the loan amount, interest rate and maturity respectively our parameters of interest can be defined as,

$$
\begin{aligned}
\eta^{r} & \equiv \frac{\partial \mathbb{E}[\log Q \mid R=r, T=\tau, X=x]}{\partial \log r} \\
\eta^{\tau} & \equiv \frac{\partial \mathbb{E}[\log Q \mid R=r, T=\tau, X=x]}{\partial \log \tau} .
\end{aligned}
$$

where $X$ is a vector of other observed variables that might affect loan demand.
The exact expression of the structural loan demand $Q(r, \tau, x, u)$ will be a complex function of interest rate $r$, maturity $\tau$ and other variables both observable $x$ and unobservables $u$. In practice, as in Argyle et al. (2020), we will be estimating the following log-linear approximation of loan demand:

$$
\begin{equation*}
\log q_{i j g t}=\eta^{r} \log r_{i j g t}+\eta^{\tau} \log \tau_{i j g t}+x_{i j g t}^{\prime} \beta+u_{i j g t} \tag{12}
\end{equation*}
$$

where $q_{i j g t}, r_{i j g t}$ and $\tau_{i j g t}$ are respectively the loan size, interest rate and maturity of mortgage $i$ originated by lender $j$, in geographic market $g$ and in period $t$. The vector $x_{i j g t}$ contains exogenous variables that affect loan demand which might also include various type of fixedeffects.

The basic identification challenge in estimating equation (12) is that loan size $q$, interest rate $r$ and maturity $\tau$ are jointly determined and thus both $r$ and $\tau$ are likely to be endogenous in equation (12). Before discussing how we tackle this identification challenge, it is useful to look at some data correlations. Figure 2 shows a binned scatter of the APR against loan size after partialling out maturity, interest rate type (i.e., FRM vs. ARM) together with lender, province and time fixed effects. The relationship is downward sloping; prices and quantities are negatively correlated which is reassuring as we expect loan demand to be decreasing in interest rate.

Figure 3 presents a similar scatter plot for the relationship between maturity and loan size. In this case the relationship is upward sloping; larger loans also have longer maturities consistent with the presence of liquidity constraints. As discussed in Section 3 , if every borrowers was liquidity unconstrained we would expect a flatter relationship between loan size and contract duration - in the model, unconstrained borrowers do not respond to changes in the maximum available maturity.

Before discussing the identification strategy, it is worth pointing out that while suggestive Figure 3 only tells us that larger loans are also longer. One important thing that is missing is housing consumption e.g., how housing consumption varies along the distribution of maturities? Are borrowers with longer contracts also buying bigger houses or the pattern in Figure 3 is purely debt driven? Unfortunately, our administrative data on realized mortgage contracts does not provide information on housing values. Nonetheless, we have evidence suggesting that borrowers with longer contracts are on average more indebted. In Figure 4 we use data from the Survey on Italian Household Income and Wealth (SHIW) to construct a binned scatter plot for the relationship between LTV and maturity. The pattern is clear: borrowers with longer mortgages issue more debt relative to their housing consumption. Thus, we believe that the pattern in Figure 3 is mostly driven by relatively more indebted and constrained borrowers rather than by borrowers that holding everything constant prefer bigger houses.

### 5.1 Identification

As briefly mentioned above, the estimation of equation (12) is challenging because both the APR $r$ and the contract maturity $\tau$ are jointly endogenous. In this section we briefly overview how lenders price their mortgages through the MutuiOnline.it platform and then describe how to exploit data on online offers to isolate exogenous variation in these two contract terms.

As discussed in section 4.2, for a given potential borrower profile in a given month and province we observe the APR and the net interest rate each lender is willing to offer through the MutuiOnline.it platform. Mortgage pricing decisions are typically taken at the beginning of each month when lenders submit to the platform their credit algorithm that accepts or rejects an application and, in the former case, determines the offered APR. ${ }^{21}$ When borrowers search for loans through the platform their applications do not reach lenders directly, instead it is MutuiOnline.it itself that calls lenders' credit algorithm to generate the mortgage offers (Michelangeli et al. (2020)). These offers represents lenders' pre-approval decisions and all contract terms are binding conditional on the information provided by borrowers being cor-

[^9]rect. This latter institutional feature is crucial because it ensures that posted contract terms are informative about lenders pricing policies rather than some teasing policies aimed only at attracting consumers (Carella and Michelangeli (2021)). In practice, it turns out that online posted rates are extremely good predictors for the realized mortgage rates (Carella et al. (2020)), even though the market share of mortgages finalized through MutuiOnline.it is quite small.

Next, we discuss how we use online offers to identify our parameters of interest: loan demand elasticities to interest rate and maturity. The way in which our data is constructed allows us to fix credit demand (i.e., hold a borrower profile constant), and observe how offered APR and net interest rate vary across lenders, provinces (i.e., markets) and time periods. By construction this variation reflects changes in banks' loan supply policies (e.g., how different lenders price the same borrower differently and/or how the same lender prices the same borrower differently across markets and provinces) and it is precisely what allows us to identify loan demand elasticities to interest rate and maturity. More precisely, this data allows us to trace out each lender pricing rule as a function of a given borrower characteristic, overtime or across markets while holding everything else constant.

To be more specific let us denote by $\tilde{r}_{j g t}(\tau, l, f, b)$ the posted APR lender $j$ in period $t$ and province $g$ is willing to offer to a potential borrower profile defined by the tuple of characteristics $(\tau, l, f, b)$ where $\tau$ is the maturity, $l$ the loan-to-value, $f$ is the type of interest rate (floating vs. fixed) and $b$ is a vector of borrowers characteristics including age, income and employment type. The basic idea behind our approach is to match to each realized APR $r_{i j g t}$ recorded in our administrative data one of the $\operatorname{APR} \tilde{r}_{j g t}(\tau, l, f, b)$ posted online. For each realized mortgage $i$ issued by lender $j(i)$, in month $t(i)$, province $g(i)$, with type of interest rate $f(i)$ and maturity $\tau(i)$ we have $|L| \times|B|$ corresponding APRs $\left\{\tilde{r}_{j(i), g(i), t(i)}(\tau(i), l, f(i), b)\right\}_{(l, b) \in L \times B}$ that we could match. ${ }^{22}$ This is because our administrative dataset does not contain information on contracts' LTV and borrowers' characteristics such as the ones enclosed in $b$.

In order to obtain a 1-to-1 matching we proceed in two steps. First, we average posted rates over borrowers characteristics in $b$ and denote this average as $\tilde{r}_{j g t}(\tau, l, f) \equiv|B|^{-1} \sum_{b} \tilde{r}_{j g t}(\tau, l, f, b)$. As Figure 5 shows this averaging is mostly innocuous because borrowers characteristics such as the ones enclosed in $b$ have almost no predictive power in explaining variation in offered APRs. Second, to each realized contract $i$ with APR $r_{i j g t}$ we associate the closest posted rate $\tilde{r}_{j(i), g(i), t(i)}(\tau(i), f(i), l(i))$ among the $|L|$ remaining i.e.,

$$
l(i)=\arg \min _{l}\left|r_{i j g t}-\tilde{r}_{j(i), g(i), t(i)}(\tau(i), f(i), l)\right| .
$$

[^10]Alternatively, we could have averaged posted rates over both LTV values $l$ and borrower characteristics $b$ instead of taking the closest posted rate among the $|L|$ remaining ones. While results are robust to either possibilities, we opted for the latter because it is informative about the unobserved LTV of the realized contracts (e.g., $l(i)$ is an estimate of contract $i$ 's LTV) which as shown in Figure 5 matters for mortgage pricing ${ }^{23}$ Overall, for each contract $i$ we denote by $\tilde{r}_{i j g t}$ the corresponding online offered APR.

Our final goal is to identify elasticities $\eta^{r}$ and $\eta^{\tau}$ in equation (12). To do so we use posted APRs $\tilde{r}_{i j g t}$ as instrument for the realized APR $r_{i j g t}$ and the per-euro posted monthly payment $\tilde{m}_{i j g t}$ as instrument for the realized maturity $\tau_{i j g t}$. The latter is a non-linear function of the posted APR net of fees which we can easily recover from our data on online posted offers:

$$
\begin{equation*}
\tilde{m}_{i j g t} \equiv \frac{\tilde{r}_{i j g t}-\mathrm{fee}_{i j g t}}{1-\left(1+\tilde{r}_{i j g t}-\mathrm{fee}_{i j g t}\right)^{-\tau_{i}}} . \tag{13}
\end{equation*}
$$

For these two instruments to work properly we need, after conditioning on $x_{i j g t}$, both $\tilde{r}_{i j g t}$ and $\tilde{m}_{i j g t}$ to be
i. uncorrelated with $u_{i j g t}$ and
ii. correlated with $r_{i j g t}$ and $\tau_{i j g t}$ respectively.

Condition (i.) cannot be tested but we believe it holds in our setting for at least two reasons. First, the online mortgage market is quite small and is likely to be independent from the main market ${ }^{24}$ For instance, condition (i.) would hold if demand shocks across the two markets were to be independent and the correlation between online posted rates and realized posted rates was mostly driven by supply shocks. ${ }^{25}$ In our setting this assumption is not unreasonable because market conditions were quite stable between 2018 and 2019 and there was no relevant aggregate demand shock. Second, our analysis is based on high frequency data (e.g., monthly) over a short and (economically) stable period of time (e.g., a bit more than a year) and thus the observed variation in posted rates is likely to be a consequence of high frequency changes in lending costs such as monthly fluctuations of market rates rather than changes in mortgage

[^11]demand ${ }^{26}$
Turning to condition (ii.) we expect the average posted APR to be correlated with the realized APR via supply shocks. Similarly, posted payments should be correlated with realized maturities as we expect borrowers to choose the duration of their contract depending on the offered flow of payments (Bachas (2019)). To see this recall that in the simple theoretical framework we developed in section 3 the optimal choice of maturity solves:
\[

$$
\begin{equation*}
\arg \min _{s \leq \tau} \mathbb{E}_{0} \int_{0}^{s} e^{-\rho t} u^{\prime}\left(y_{t}-b_{0}^{*}(s)\right) m(r, s) d t+\mu m(r, s) \tag{14}
\end{equation*}
$$

\]

The duration chosen clearly depends on the payments $m(r, \tau)$ so that exogenous shifts in $m$ would shift the choice of contract maturities.

### 5.2 Estimates of mortgage demand elasticities

This section aims at presenting the main empirical findings of the paper. To start, we discuss the dataset we will use to estimate our parameters of interest. This dataset merges the administrative data on realized mortgage contracts (Section 4.1) with data on online mortgage offers (Section 4.2) in order to match each realized contract with one of the posted contract available on the MutuiOnline.it platform (Section 5.1). The matching procedure reduces our sample size because some lenders do not have agreements with the MutuiOnline.it platform and thus do not post offers online. Nonetheless, in the resulting dataset, the summary statistics for the main variables do not display any substantial change.

Table 3 replicates Table 1 for our final dataset. Around $83 \%$ of our observations are successfully merged, which is consistent with MutuiOnline.it working with the largest lenders, accounting for about $80 \%$ of total mortgage originations (Carella and Michelangeli (2021)). Comparing the summary statistics of some of the main variables across Table 1 and Table 3 we can see that our merging procedure did not substantially change these distributions. The most affected one is the distribution of loan sizes: the average amount decreased by more or less 700 euros and the overall dispersion is slightly lower. Other than that, the summary statistics shown in Tables 1 and 3 look similar, which is reassuring.

We are now ready to outline the results from the 2SLS strategy we employ to estimate equation (12). As discussed in section 5.1 we will use online posted rates $\tilde{r}_{i j g t}$ to instrument for realized APR $r_{i j g t}$ and posted monthly payments $\tilde{m}_{i j g t}$ to instrument for realized contract maturities $\tau_{i j g t}$. The corresponding first stage results are shown in Table 4 , where each column

[^12]corresponds to the first stage regression of one of the two endogenous regressors. The left column highlights that the realized APR is, as expected, positively correlated with the posted APR while it is negatively correlated (although non-significantly) with posted monthly payments. On the other hand, the right column shows that the realized maturity is positively correlated with posted APR and negatively correlated with the posted monthly payments. These signs are intuitive as lenders usually charge more for longer contracts (e.g., mortgages yield curves are upward sloping) and longer contract typically entail lower payments. Both first stage regressions in table 4 include additional controls such as whether the interest rate is fixed or floating together with province, month and lender fixed effects. Province fixed effects should capture local and time-invariant housing market conditions, month fixed effects should soak up any aggregate shock to the housing market and lender fixed effects should take into account any unobserved time-invariant product quality. Finally, both first stage regressions report the Sanderson-Windmeijer partial F-statistics testing for weak identification of each endogenous regressor separately. Both statistics are way above 10 which suggests that our instruments are not weak.

The first column of Table 6 reports the 2SLS results for our baseline specification. The estimates for the loan demand elasticity to interest rate are negative, significant and around $\eta^{r} \approx-0.22$. The sign and significance of these estimates are reassuring as we typically think about credit demand as being downward sloping in the interest rate. In terms of magnitude, our estimates suggest that credit demand is rather inelastic in the sense that a $1 \%$ increase in interest rate reduces the average loan size by less than $1 \%$. This seems to be consistent with interest rate elasticities of loan demand estimated in the recent auto-loan literature. Argyle et al. (2020) find that a $1 \%$ increase in interest rate reduces car loan size by more or less $0.17 \%$ whereas in our setting we estimate mortgage size would decrease by more or less $0.22 \%$. The fact that credit demand is more elastic for mortgages than for auto-loans is reasonable as monthly payments are more responsive to interest rate for loans with longer maturities. For the US mortgage market DeFusco and Paciorek (2017) find that a 100 bp increase in mortgage rate would lead to a decline in mortgage demand of more or less $3 \%$ which, at the average mortgage rate for that period, roughly implies an interest rate elasticity quite similar to ours and around $-0.19 \cdot{ }^{27}$

Turning to maturity elasticities, our estimates suggest that a $1 \%$ increase in maturity increases loan size by more or less $\eta^{\tau} \approx 0.30$ percent. This significant positive relationship between loan size and maturity suggests that loan demand is upward sloping in the mortgage duration which is, as discussed in Section 3, consistent with the average borrower being liquidity constrained. For the US auto-loan market Argyle et al. (2020) estimate a positive and

[^13]significant maturity elasticity of car-loan demand, in magnitude few times larger than our estimates (e.g., around 0.66). This discrepancy is reasonable for at least two reasons: first, car loans are shorter than mortgages which implies that monthly payments (and thus constrained borrowers' demand) will be more sensitive to changes in contract maturity; second, car loans are substantially smaller in size than mortgages which tends to artificially inflate estimated demand elasticities.

### 5.3 Short vs. Long term borrowers

Do liquidity constrained borrowers prefer short or long term loans? Theoretically, the answer is straightforward: constrained borrowers should prefer long term loans. The simple framework developed in Section 3 offers a clear intuition: longer contracts have lower payments and thus allow constrained households to borrow more and increase current consumption (equation (10). Overall, whenever possible, we expect constrained borrowers to select into longer maturities.

To answer the question empirically we would need to proxy for liquidity constraints. A natural way to identify liquidity constrained borrowers would be ranking them based on their income or wealth (Attanasio et al. (2008)). Unfortunately, our administrative data do not provide us with this information which makes us unable to directly test whether wealthier borrowers are more likely to choose shorter maturities. Nonetheless, we provide survey evidence that this is likely to be true. Figure 6shows a binned scatter plot of households wealth against their mortgage maturity for the sample of borrowers interviewed in the SHIW and, as expected, wealthier borrowers are more likely to have shorter mortgages ${ }^{28}$

Based on the above discussion, in this section we replicate the empirical analysis presented in Section 5.2 after splitting our sample between short and long term borrowers. If short term borrowers are more likely to be wealthier and unconstrained, we should expect to find lower maturity elasticities in the subsample of short-term borrowers.

Table 5 compares the summary statistics of the two groups. Compared to long term loans, short term loans are on average cheaper, slightly smaller and, importantly, require substantially higher monthly payments which presumably makes them affordable only to wealthier borrowers. Indeed, the average monthly payment for loans with maturities less than or equal to 15 years is around 980 euros, $66 \%$ higher than the average payment for loans with maturities above 15 years.

The 2SLS estimates for both subsamples are reported in the last two columns of Table 5 .

[^14]The results are striking: the estimated maturity elasticity of loan demand for short term borrowers is not significantly different from zero, consistent with short term borrowers being unconstrained and thus unresponsive to their mortgage duration. Long term borrowers' estimated maturity elasticity is instead positive, significant and $30 \%$ larger than the maturity elasticity estimated for the full sample of borrowers which is consistent with long term borrowers being on average more liquidity constrained and thus more sensitive to changes in their contract maturities. Interest rates elasticities between the two groups are also slightly different. For long term borrowers estimates are similar to the ones for the full sample i.e, around -0.26 , whereas for short term borrowers the estimated APR elasticity is, although negative and significant, a bit lower i.e., about -0.15 . This discrepancy is in part artificial and can be attributed to the fact that interest rates charged on shorter contracts are on average lower. Indeed, at the average APR of 1.78 and 2.36 for short and long-term borrowers respectively, a $1 \%$ increase in interest rate reduces loan demand by $8.5 \%$ and $11 \%$ respectively so that the interest rate sensitivity of credit demand is, in relative terms, similar across the two groups.

## 6 Interpretation and Policy Implications

Two main conclusions can be drawn based on the elasticities estimates we presented in Section 5. First, aggregate mortgage demand is significantly upward sloping in loan maturity which suggests that liquidity constraints are binding for the average borrower. In Section 5.2 we estimated that a $1 \%$ increase in maturity would push up aggregate credit demand by more or less $0.30 \%$. Second, this relationship between loan size and contract maturity is also significantly convex which means that long term borrowers are the ones driving aggregate credit demand up as maturity increases. The results in Section 5.3 highlight that short term borrowers are unresponsive to their contract maturity while a $1 \%$ increase in duration increases long term borrowers' demand by $0.40 \%$. This is consistent with short term borrowers being less constrained and wealthier than long term borrowers.

### 6.1 Estimates interpretation

To gain intuition it is useful to put some numbers on these estimates. Take for example a borrower with an outstanding 20 years mortgage of about 140,000 euros with a $2.5 \%$ APR. What would have happened if this borrower were offered a longer mortgage at origination of 30 years instead of 20 ? Given our maturity elasticity estimate, for the average borrower the amount borrowed would have been $15 \%$ higher (e.g., around 160,000 euros). This increase in demand is in magnitude reasonably close to a commensurable decrease in interest rate; holding
maturity constant, a reduction in interest rate from $2.5 \%$ to $1.25 \%$ would increase the loan size from 140,000 to roughly 156,000 euros. Overall, borrowers respond to both maturity and interest rate which from the lens of our simple life-cycle model with long-term debt (Section (3) is indicative of the presence of liquidity constraints.

Another important insight that stems from the discussion in Section 5.3 is that credit demand response to changes in maturity is heterogeneous. Short term borrowers are unresponsive to the duration of their contract while long-term borrowers are even more sensitive to changes in maturity compared to the average borrower. On the other hand, short and long term borrowers' response to changes in interest rate is similar. As discussed in Section 5.3, at the average loan amount of 140,000 euros a 1 percentage point increase in APR increases the loan amount by 12,000 and 15,000 euros respectively for short and long-term borrowers. The fact that maturity elasticities, as opposed to interest rate elasticities, are heterogeneous has important policy implications to which we turn next.

### 6.2 Policy Implications

What are the policy implications of our findings? One immediate implication of our estimates is that policy interventions targeting the supply of maturity might have a substantial impact on household credit demand. In terms of magnitude our estimates suggest that these interventions would have a similar (or possibly stronger) impact than policies targeting interest rates: holding everything else constant, policies that incentivize lenders to issue longer mortgages would increase credit demand at least as much as policy that produce a commensurable reduction in interest rates.

While our elasticity estimates suggest that both maturity and interest rate policies have an impact on aggregate credit demand, a natural question is: from whom this additional demand is coming? According to our estimates the answer differs depending on the type of policy. A reduction in interest rates will boost credit demand of both constrained and unconstrained borrowers whereas an increase in the supply of longer contracts will increase loan demand only for constrained borrowers.

This differential in the composition of the aggregate demand response creates important trade-offs when comparing interest rate and maturity policies. Policies targeting the supply of maturities affect aggregate demand mostly by increasing credit accessibility and thus might be better suited when the goal is to increase credit access without directly affecting the relative price of borrowing. On the other hand, a possible concern with policies that incentivize lenders to issue longer mortgages is that the aggregate level of indebtedness might rise. The pattern in Figure 4 highlights that households with longer mortgages tend to have higher loan-to-values.

This suggests that an increase in the supply of longer mortgages is likely to increase aggregate debt more than aggregate housing consumption which in turn can lead to an increase in the level of aggregate systemic risk and a decrease in the share of aggregate home equity as longer loans amortize at a slower pace.

To sum up, our empirical findings highlight that household debt maturity could be an important dimension to consider when evaluating the real effects of credit supply shocks. For instance, interventions that tilt the yield curve (e.g., the Maturity Extension Program) might provide an additional transmission channel of unconventional monetary policies to the household sector.

## 7 Conclusion

This paper joins a relatively recent literature that assess the importance of maturity elasticity of loan demand in credit markets. We study how mortgage demand responds to changes in interest rate and maturity and find that while (as expected) demand decreases in interest rate, it is significantly upward sloping in mortgage maturities. This is consistent with the average borrower being liquidity constraint- if offered a longer loan the average borrower would borrow more to increase current consumption.

To overcome the endogeneity of interest rate and maturity in the estimation of loan demand we use variation in online posted rates and posted monthly payments as instruments for both realized interest rates and realized contract maturities. Our 2SLS elasticity estimates emphasize two important facts. First, on average borrowers are responsive to changes in both interest rate and maturity. A $1 \%$ increase in the latter increases loan demand by $0.30 \%$ which is in magnitude close to a $1 \%$ increase in interest rate which reduces loan demand by $0.22 \%$. Second, this high maturity elasticity turns out to be driven by long-term borrowers while short-term borrowers are unresponsive to changes in their contract length. This evidence is consistent with short-term borrowers being on average wealthier and able to afford the higher payments that typically accompany short-term contract, as opposed to long-term borrowers who are instead likely to be liquidity constrained.

In terms of policy our results underline the importance of debt maturity as an additional dimension that policy makers can leverage when designing interventions aimed at affecting aggregate demand through credit market access. For instance, monetary policy interventions aimed at flattening the yield curve are likely to affect credit demand both because longer contracts are cheaper (interest rate channel) but also because lenders have more incentives to offer longer contracts which increases credit access by relaxing borrowers' liquidity constraints (maturity channel). Overall, our elasticity estimates call up for a more thorough analysis of the role
of the maturity channel of credit supply shocks which, as of now, has been largely ignored.

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## 8 Figures



Figure 1: Distribution of maturities for mortgages issued in Italy between 2018 and 2019 conditional on the loan amount being in between 119,000 and 121,000 thousands euros. Maturities are recovered from the CR and TAXIA administrative datasets as described in Appendix A. Source: CR and TAXIA.


Figure 2: Binned scatter plot of realized APR against loan size after partialling out maturity, type of rate (FRM vs. ARM) and lenders, province and month fixed effects. Source: CR and TAXIA.


Figure 3: Binned scatter plot of realized loan size against maturity after partialling out APR, type of rate (FRM vs. ARM) and lenders, province and month fixed effects. Source: CR and TAXIA.


Figure 4: Binned scatter plot of LTV against mortgage maturity after partialling out borrowers' age, wealth, mortgage rate and type of rate (FRM vs. ARM) and time of origination fixed effects. Source: SHIW waves 2002-2016.

Offered APR variation decomposition


Figure 5: Explained sum of squared decomposition of online offered APR by contract characteristics, borrower characteristics and profile characteristics (i.e., the combination of the previous two). Source: MutuiOnline.it.


Figure 6: Binned scatter plot of household wealth against mortgage maturity after partialling out borrowers' age, mortgage rate, type of rate (FRM vs. ARM), loan size and time of origination fixed effects. Wealth is measured before mortgage origination. Source: SHIW waves 2002-2016.

|  | Obs. | Mean | Std. Dev. | Q1 | Q2 | Q3 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| APR (\%) | 186424 | 2.32 | 0.77 | 1.86 | 2.24 | 2.73 |
| Loan amount (€) | 186424 | 136818.93 | 59767.37 | 100000.00 | 120000.00 | 155630.00 |
| Monthly Payment (€) | 186424 | 628.15 | 304.82 | 448.61 | 552.18 | 708.49 |
| Maturity (years) | 186424 | 23.53 | 5.68 | 20.01 | 25.00 | 29.88 |
| FRM | 186424 | 0.72 | 0.45 | 0.00 | 1.00 | 1.00 |

Table 1: Summary statistics for mortgages issued in Italy between March 2018 and August 2019. Source: CR and TAXIA.
FIXED RATE OFFERS

|  | Maturity (years) |  |  |  | LTV |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 10 | 15 | 20 | 30 | 50 | 60 | 80 | 85 |
| APR (\%) | $\begin{gathered} 1.70 \\ (0.52) \end{gathered}$ | $\begin{gathered} 1.93 \\ (0.51) \end{gathered}$ | $\begin{gathered} 2.05 \\ (0.50) \end{gathered}$ | $\begin{gathered} 2.39 \\ (0.45) \end{gathered}$ | $\begin{gathered} 1.85 \\ (0.43) \end{gathered}$ | $\begin{gathered} 1.94 \\ (0.42) \end{gathered}$ | $\begin{gathered} 2.09 \\ (0.43) \end{gathered}$ | $\begin{gathered} 4.21 \\ (0.20) \end{gathered}$ |
| Net Interest Rate (\%) | $\begin{gathered} 1.39 \\ (0.49) \end{gathered}$ | $\begin{gathered} 1.70 \\ (0.48) \end{gathered}$ | $\begin{gathered} 1.84 \\ (0.47) \end{gathered}$ | $\begin{gathered} 2.21 \\ (0.42) \end{gathered}$ | $\begin{gathered} 1.62 \\ (0.45) \end{gathered}$ | $\begin{gathered} 1.72 \\ (0.44) \end{gathered}$ | $\begin{gathered} 1.86 \\ (0.43) \end{gathered}$ | $\begin{gathered} 3.83 \\ (0.18) \end{gathered}$ |
| Monthly Payment | $\begin{gathered} 0.89 \\ (0.02) \end{gathered}$ | $\begin{gathered} 0.63 \\ (0.02) \end{gathered}$ | $\begin{gathered} 0.50 \\ (0.02) \end{gathered}$ | $\begin{gathered} 0.38 \\ (0.02) \end{gathered}$ | $\begin{gathered} 0.60 \\ (0.19) \end{gathered}$ | $\begin{gathered} 0.60 \\ (0.19) \end{gathered}$ | $\begin{gathered} 0.60 \\ (0.19) \end{gathered}$ | $\begin{gathered} 0.70 \\ (0.20) \end{gathered}$ |
| ADJUSTABLE RATE OFFERS |  |  |  |  |  |  |  |  |
|  | Maturity (years) |  |  |  | LTV |  |  |  |
|  | 10 | 15 | 20 | 30 | 50 | 60 | 80 | 85 |
| APR (\%) | $\begin{gathered} 1.21 \\ (0.48) \end{gathered}$ | $\begin{gathered} 1.15 \\ (0.47) \end{gathered}$ | $\begin{gathered} 1.14 \\ (0.46) \end{gathered}$ | $\begin{gathered} 1.26 \\ (0.47) \end{gathered}$ | $\begin{gathered} 1.06 \\ (0.45) \end{gathered}$ | $\begin{gathered} 1.17 \\ (0.42) \end{gathered}$ | $\begin{gathered} 1.26 \\ (0.42) \end{gathered}$ | $\begin{gathered} 2.45 \\ (0.07) \end{gathered}$ |
| Net Interest Rate (\%) | $\begin{gathered} 0.91 \\ (0.45) \end{gathered}$ | $\begin{gathered} 0.93 \\ (0.45) \end{gathered}$ | $\begin{gathered} 0.96 \\ (0.45) \end{gathered}$ | $\begin{gathered} 1.12 \\ (0.45) \end{gathered}$ | $\begin{gathered} 0.85 \\ (0.44) \end{gathered}$ | $\begin{gathered} 0.96 \\ (0.41) \end{gathered}$ | $\begin{gathered} 1.06 \\ (0.41) \end{gathered}$ | $\begin{gathered} 2.14 \\ (0.10) \end{gathered}$ |
| Monthly Payment | $\begin{gathered} 0.87 \\ (0.02) \\ \hline \end{gathered}$ | $\begin{gathered} 0.59 \\ (0.02) \end{gathered}$ | $\begin{gathered} 0.46 \\ (0.02) \end{gathered}$ | $\begin{gathered} 0.33 \\ (0.02) \\ \hline \end{gathered}$ | $\begin{gathered} 0.56 \\ (0.20) \end{gathered}$ | $\begin{gathered} 0.57 \\ (0.20) \end{gathered}$ | $\begin{gathered} 0.57 \\ (0.20) \end{gathered}$ | $\begin{gathered} 0.62 \\ (0.20) \end{gathered}$ |

Table 2: Mean and standard deviations (across lenders, time, provinces and borrower profiles) of online offered terms by maturity and LTV. Posted monthly payments are per $€ 100$ borrowed. Standard deviations are in parenthesis. Source: MutuiOnline.it.

|  | Obs. | Mean | Std. Dev. | Q1 | Q2 | Q3 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| APR (\%) | 154590 | 2.32 | 0.78 | 1.86 | 2.24 | 2.73 |
| Loan amount (€) | 154590 | 136143.59 | 58920.30 | 99200.00 | 120000.00 | 155000.00 |
| Monthly Payment (€) | 154590 | 621.08 | 297.74 | 445.41 | 547.86 | 700.81 |
| Maturity (years) | 154590 | 23.68 | 5.71 | 20.01 | 25.02 | 29.89 |
| FRM | 154590 | 0.74 | 0.44 | 0.00 | 1.00 | 1.00 |

Table 3: Summary statistics for mortgages issued in Italy between March 2018 and August 2019 after merging the data on online offers. Source: CR, TAXIA and MutuiOnline.it.

|  | APR | $\log ($ Maturity $)$ |
| :--- | :---: | :---: |
| posted APR | $1.026^{* * *}$ | $0.0011^{* * *}$ |
|  | $(0.069)$ | $(0.000)$ |
| posted monthly payment | $-54.64^{*}$ | $-1.952^{* * *}$ |
|  | $(23.97)$ | $(0.0247)$ |
| Observations | 150502 | 150502 |
| Sanderson-Windmeijer partial F-statistic | 165.5 | 329.8 |
| partial R-squared | 0.369 | 0.896 |
| Controls | Yes | Yes |
| Fixed Effects |  |  |
| - time | Yes | Yes |
| - province | Yes | Yes |

Table 4: First stage results for the two endogenous regressors in equation (12). Posted monthly payment is per $€ 100$ borrowed. Controls include type of interest rate (fixed vs. floating), borrowers' age, gender and nationality. Standard errors in parentheses are clustered at the lender level. $* \mathrm{p}<0.05{ }^{* *} \mathrm{p}<0.01$ *** $\mathrm{p}<0.001$.
SHORT TERM BORROWERS

|  | Obs. | Mean | Std. Dev. | Q1 | Q2 | Q3 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| APR (\%) | 11243 | 1.78 | 0.69 | 1.30 | 1.67 | 2.16 |
| Loan amount (€) | 11243 | 123934.56 | 61835.79 | 88498.00 | 100037.00 | 136000.00 |
| Monthly Payment (€) | 11243 | 980.88 | 518.89 | 694.97 | 851.46 | 1086.81 |
| Maturity (years) | 11243 | 11.68 | 2.25 | 10.00 | 10.01 | 14.54 |
| FRM | 11243 | 0.84 | 0.37 | 1.00 | 1.00 | 1.00 |
| LONG TERM BORROWERS |  |  |  |  |  |  |
| APR (\%) | Obs. | Mean | Std. Dev. | Q1 | Q2 | Q3 |
| Loan amount (€) | 143347 | 2.36 | 0.77 | 1.92 | 2.27 | 2.76 |
| Monthly Payment (€) | 143347 | 137101.17 | 58578.21 | 100000.00 | 121126.00 | 156000.00 |
| Maturity (years) | 143347 | 592.86 | 252.06 | 437.09 | 533.91 | 668.93 |
| FRM | 143347 | 0.73 | 4.75 | 20.02 | 25.03 | 29.92 |

Table 5: Summary statistics for mortgages issued in Italy between March 2018 and August 2019 by borrower type. Short term borrowers are the ones with mortgages with duration lower or equal than 15 years. Source: CR, TAXIA and MutuiOnline.it.

|  | Baseline | Short-term Borrowers | Long-terms borrowers |
| :--- | :---: | :---: | :---: |
| $\log (\mathrm{APR})$ | $-0.220^{* * *}$ | $-0.154^{* * *}$ | $-0.257^{* * *}$ |
|  | $(0.0143)$ | $(0.0180)$ | $(0.00900)$ |
| $\log$ (Maturity) | $0.293^{* * *}$ | 0.00473 | $0.391^{* * *}$ |
|  | $(0.0111)$ | $(0.0370)$ | $(0.0155)$ |
| Observations | 150502 | 11026 | 139476 |
| Controls <br> Fixed Effects | Yes | Yes | Yes |
| - time | Yes | Yes |  |
| - province | Yes | Yes | Yes |

Table 6: Results of 2SLS estimation of equation (12) for the whole sample (1st columns), shortterm borrowers ( 2 nd column) and long-term ( 3 rd columns). Short-term borrowers are defined as the ones with mortgages with maturity lower or equal than 15 years. Controls include type of interest rate (fixed vs. floating), borrowers' age, gender and nationality. Standard errors in parentheses are clustered at the lender level. * $\mathrm{p}<0.055^{* *} \mathrm{p}<0.01$ *** $\mathrm{p}<0.001$.

## A Appendix: Recovering contract maturities

After merging CR and TAXIA dataset for each mortgage contract $i$ we have the following information:

- $j$ : the lender identifier
- $L$ : the loan size
- $o$ : the month of origination
- $r$ : the interest rate gross of fees (APR)
- $P_{m}$ : the outstanding principal at month $m \geq o$
- $I_{q}$ : the euros amount of interest paid in quarter $q \geq q(o)$

To recover the maturity of contract $i$ we will rely on the french amortization lenders use to compute monthly payments. To do so we first need the interest rate net of fees $\rho$ which we can recover for each quarter $q$ as follows:

$$
\rho_{q}=\frac{I_{q}}{P_{q}}
$$

where $P_{q}$ is the total outstanding principal in quarter $q$. Second, for each quarter $q$, we the equal sized monthly payment:

$$
M_{q}=\frac{I_{q}+C_{q}}{|\{m: q(m)=q\}|}
$$

where $C_{q}=\sum_{m: q(m)=q}\left(P_{m-1}-P_{m}\right)$ is the amount of principal repaid in quarter $q 2^{29}$
The maturity of the contract can be then recovered by inverting the amortization function:

$$
\tau_{q}=\frac{\log \left(\frac{M_{q}}{M_{q}-L \rho_{q}^{12}}\right)}{\log \left(1+\rho_{q}^{12}\right)}
$$

where $\rho_{q}^{12} \equiv \rho_{q} / 12$ is the monthly monthly mortgage rate net of fees in quarter $q$. In practice, this procedure gives us a measure of the maturity for each quarter in which we observe contract $i$ 's outstanding balance. For FRM this is basically constant while for ARM the resulting maturity can vary because $\rho_{q}$ changes overtime. This is especially true for ARM contracts with a constant monthly payment which, according to the SHIW, is the most common option selected by households opting for a floating contract. As a measure of contract maturity we pick the one computed for the most recent quarter in which we observe the contract balance.

[^15]
## B An empirical model of search and maturity choice

Another way in which households can reduce the cost of their loan is by visiting different lenders with the hope that some of them might offer better financial conditions. Gains from searching across different lenders are not negligible in this market. Figure 7 suggests that the residual dispersion in the equilibrium interest rates is, even when comparing similar contracts in the same market, quite substantial. An average borrower can save up to 200 basis points by shopping across different lenders in the same province and quarter. A similar conclusion can be drawn when looking at offered rates. Figure 8 suggests that the residual dispersion in the offered interest rates is, even when comparing similar contracts and similar borrowers, again substantial. An average borrower can save up to 100 basis points by choosing the cheapest lender ${ }^{30}$

Even though gains from search seem to be first order, to rationalize the dispersion in rates as an equilibrium outcome we must consider the possibility that informational frictions are also relevant in this market. The existence of heterogeneous search costs that prevent households from finding the best available offer has been widely documented in the IO and Household Finance literature. ${ }^{31}$ The presence of these search frictions gives lenders market power even though they sell an homogeneous product and in turn generates equilibrium price dispersion. The latter is what allows the econometrician to identify the distribution of search costs and assess the extent of lenders market power ${ }^{32}$

In this appendix we develop a structural model of mortgage search in which we also allow borrowers to optimally select the term of their contract from a discrete set of maturities. Each lender posts its own mortgage yield curve i.e., an interest rate for each of the possible maturities. When a borrower visits a lender she observes the entire yield curve offered and picks the combination of interest and maturity that gives her the highest utility. Given the highest utility obtained in previous searches our borrower then decides whether or not to pay a constant search cost to visit an additional lender or stop and sign for the best loan among the quotes previously gathered.

We show how the observed joint distribution of interest rates and maturities can be used to inform the structural parameters of the model. The shares of contract at each maturity identify borrowers sensitivity to periodic loan payments separately from their sensitivity to interest

[^16]

Figure 7: APR distribution after controlling Figure 8: Offered APR distribution after confor amount borrowed, type of rate, maturity, trolling for LTV, maturity, type of rate, inquarter and province fixed effects.
come, age, job type, month and province.
rates. This allows us to quantify consumers willingness to pay for smoothing loan payments for an additional period.

Turning to search costs, we show that on average search costs are, by construction, less dispersed compared to the case in which borrowers are assumed to be responsive to interest rate only or equivalently, when the periodic payment does not enter consumers' indirect utility together with the other contract characteristics. This is a consequence of the amortizing feature of these type of contracts which creates a wedge between the distribution of rates and the corresponding distribution of payments that borrowers face in the market. In subsection B.5 of this Appendix we prove that, conditional on a given maturity, the distribution of periodic payments is relatively less dispersed than the corresponding distribution of interest rates. Therefore, when borrowers are more sensitive to loan payments it is the distribution of payments that pins down the distribution of search costs rather than the distribution of interest rates.

The structural model we develop does not apply only to mortgage search and discrete maturity choice. Lenders in the US usually offer borrowers different combinations of interest rate and discount points. When a borrower visits a lender she faces an whole menu of rate and points from which to choose. Buying more points upfront typically reduces the mortgage rate ${ }^{33}$ Our model can be used to separate borrowers sensitivity to points from interest rate sensitivity also accounting for search frictions.

Auto loans are another example in which lenders offer menus of interest rates and maturities and thus our model can be applied. On their webpage titled "How car loans work?" Bank of America advises consumers on how to choose their car loans ${ }^{34}$ "One of the most important

[^17]| Lender \# | 10 years |  |  | 15 years |  |  | 20 years |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 30 years |  |  |  |  |  |  |  |  |
|  | Rate | Payment | Rate | Payment | Rate | Payment | Rate | Payment |
| 1 | 1.2 | 884 | 1.5 | 620 | 1.65 | 489 | 2 | 369 |
| 2 | 1.33 | 890 | 1.69 | 629 | 1.85 | 498 | 2.22 | 380 |
| 3 | 1.94 | 917 | 2.3 | 657 | 2.56 | 532 | 2.85 | 413 |
| 4 | 2.24 | 930 | 2.6 | 671 | 2.76 | 542 | 2.85 | 413 |
| 5 | 1.94 | 917 | 2.3 | 657 | 2.56 | 532 | 2.85 | 413 |
| 6 | 1.25 | 886 | 1.47 | 619 | 1.4 | 477 | 1.81 | 360 |
| 7 | 1.15 | 882 | 1.55 | 622 | 1.55 | 484 | 2.2 | 379 |

Table 7: Set of yields curves offered to an household looking for a fixed rate mortgage of euros 100,000, LTV $=60$, income of euros 3000 in the province of Milan. Source: MutuiOnline.it.
things to understand about how auto loans work is the relationship between the loan term and the interest you pay. A longer loan term can dramatically lower your monthly payment, but it also means you pay more in interest... Some people will benefit most by taking a longer term to reduce monthly payments and using the difference to pay down higher-interest debt. Others will prefer to make a higher monthly payment and pay off the loan sooner."

Empirically, endogenizing the loan term choice would allow us to identify payments sensitivity separately from rate sensitivity by exploiting the observed shares of contract maturities. This model also highlights the fact that borrowers might be willing to accept higher interest rates not because it is costly to find better quotes but because their high sensitivity to payments pushes them to select long term maturities. Thus, when comparing quotes from different lenders they will be mostly interested in how much their periodic payments reduces rather than how much lower is the interest rate. As we show in subsection B. 5 , for a given loan term, savings in terms of lower periodic payments are, in relative terms, smaller than savings in terms of lower interest. To see this informally, suppose the distribution of offered yield curves is the one showed in Table B. For the 20 years contract, switching from the most expensive to the cheapest lender reduces your rate by $49 \%$ while your payment goes down at most by $12 \%$. Therefore, when loan periodic payment is the borrowers' most relevant price dimension their gains from searching are lower ${ }^{35}$

[^18]
## B. 1 A model of search over menus

Setup. In this section we introduce a model of mortgage search in which borrowers are allowed to choose their loan term from a discrete set of loan maturities. In this market each lender $j$ posts its own mortgage yield curve $r_{j}=\left(r_{j t}\right)_{t=1}^{T}$ which indicates the interest rate lender $j$ charges at each of the $T$ possible loan maturities. We assume $T$ is discrete.

Borrowers are imperfectly informed: they do not know who is the lender offering the best contract terms. They are only aware of the distribution of offered interest rates at each maturity $r_{t} \sim F_{r}(r \mid t)$ for all $t \in\{1, \ldots, T\}$. In order to obtain better quotes borrowers need to visit different lenders but this is costly. Borrower $i$ needs to pay search $\operatorname{cost} c_{i}$ to obtain a draw $r_{j}$ from the market distribution of yield curves. Search costs are heterogeneous and distributed with cdf $c \sim F_{c}(c)$. After observing yield curve $r_{j}$, borrower $i$ is then able to customize its contract term and will pick maturity $t$ that maximizes its utility. Borrowers have preferences defined over the characteristics of their contract. In particular, the indirect utility of borrower $i$ choosing loan $j$ with maturity $t$ is given by $\sqrt{36}$

$$
\begin{align*}
u_{i j t} & =x_{j t} \beta-\alpha_{m} m_{j t}-\alpha_{r} r_{j t}+\varepsilon_{i j t}  \tag{15}\\
& =x_{j t} \beta-\delta_{j t}+\varepsilon_{i j t}
\end{align*}
$$

where $\delta_{j t} \equiv \alpha_{m} m_{j t}+\alpha_{r} r_{j t}, m_{j t}$ is the contract periodic payment at maturity $t, r_{j t}$ is the contract interest rate at maturity $t$ and $x_{j t}$ is a vector of other contract characteristics. ${ }^{37}$ We refer to the parameters $\alpha_{m}$ and $\alpha_{r}$ as borrowers sensitivity to periodic payments and interest rate respectively ${ }^{38}$

Borrower dynamic problem. The presence of informational frictions implies that not all components of $u_{i j t}$ are known to borrower $i$ before search. The set of characteristics $x_{j t}$ is known but the component $\delta_{j t}$ and the idiosyncratic shock $\varepsilon_{i j t}$ are unknown. Borrowers only know the distributions of $\delta_{j t} \sim F_{\delta}(\boldsymbol{\delta} \mid t)$ and $\varepsilon \sim F_{\varepsilon}(\varepsilon)$ and need to pay search cost $c_{i}$ to obtain

[^19]a draw of $\left(\delta_{j t}\right)_{t=1}^{T}$ and $\left(\varepsilon_{j t}\right)_{t=1}^{T}$. After each search borrower $i$ picks the contract maturity $t$ to maximize its utility. We denote the after search realized utility as
$$
\tilde{u}_{i j} \equiv \max _{t}\left\{u_{i j t}\right\},
$$
with $\tilde{u}_{i j}$ in hand our borrower then decides whether to stop search and sign the contract that corresponds to the highest $\tilde{u}$ searched so far or to pay $c_{i}$ and obtain an additional quote. Consumer $i$ 's Bellman equation is given by:
\[

$$
\begin{equation*}
V_{i}(\tilde{u})=\max \left\{\tilde{u},-c_{i}+\int \max \left\{V_{i}(\tilde{u}), V_{i}\left(\tilde{u}^{\prime}\right)\right\} d F_{\tilde{u}}\left(\tilde{u}^{\prime}\right)\right\} . \tag{16}
\end{equation*}
$$

\]

We emphasize that this is not a standard product search problem. The reason is that when visiting a lender our borrower is exposed to a whole menu of products (i.e., combinations of rates and maturities) from which it picks the one that maximizes its utility. Each search is thus followed by a discrete choice problem over mortgage maturities. This is embedded in the fact that the problem state variable is $\tilde{u}$ and not simply $u$. The presence of asymmetric information about who is the lender offering the best quotes together with uncertainty about the idiosyncratic shocks translates into uncertainty about what is the maximum utility $\tilde{u}$ that borrower can obtain from each lender. To solve this uncertainty borrower $i$ pays search cost $c_{i}$ to obtain sequential draws from $F_{\tilde{u}}$. This distribution of utilities values is what characterizes consumers marginal benefit from searching.

Optimal search policy. Even though borrowers' search problem is more complex than the standard search a la McCall (1970) the theoretical properties of the optimal search policy are largely unaffected. This is a consequence of the fact that the loan term choice following each search decision is assumed to be discrete. This ensures that $\tilde{u}$ is always well defined without imposing any restrictions on the yield curves offered by lenders.

Now let us consider borrower $i$ and let $\tilde{u}$ be the highest realized utility $i$ has obtained so far. Borrower $i$ will visit another lender if and only if the marginal benefit of an additional search is higher than the marginal cost of search $c_{i}$ :

$$
\begin{aligned}
\operatorname{MBS}(\tilde{u}) & \equiv \mathbb{E}_{\left(\delta_{1}, \ldots, \delta_{T}\right)}\left[\int_{\tilde{u}}^{\infty}(u-\tilde{u}) d F_{\tilde{u}}\left(u \mid \delta_{1}, \ldots, \delta_{T}\right)\right] \\
& =\int_{\left(\delta_{1}, \ldots, \delta_{T}\right)} \int_{\tilde{u}}^{\infty}(u-\tilde{u}) d F_{\tilde{u}}\left(u \mid \delta_{1}, \ldots, \delta_{T}\right) d F_{\delta}\left(\delta_{1}, \ldots, \delta_{T}\right) \\
& >c_{i}
\end{aligned}
$$

Following standard arguments it can be shown that the optimal search policy will be a reservation value policy. Borrower $i$ will keep searching until she meets a lender that gives her
an utility above some given threshold value $v_{i}^{*}$. This reservation value $v_{i}^{*}$ is implicitly defined by the following condition:

$$
\begin{equation*}
c_{i}=\int_{\left(\delta_{1}, \ldots, \delta_{T}\right)}\left(\int_{v_{i}^{*}}^{\infty}\left(u-v_{i}^{*}\right) f_{\tilde{u}}\left(u \mid \delta_{1}, \ldots, \delta_{T}\right) d u\right) f_{\delta}\left(\delta_{1}, \ldots, \delta_{T}\right) d \delta_{1} \ldots d \delta_{T} \equiv \phi\left(v_{i}^{*}\right) . \tag{17}
\end{equation*}
$$

Condition (17) tells us that $v_{i}^{*}$ is the utility value at which borrower $i$ equalizes marginal benefit of an additional search to its marginal cost. At $\tilde{u}=v_{i}^{*}$ borrower $i$ is just indifferent between visiting another lender or stop searching. This indifference condition is the typical optimal stopping condition that characterizes standard search models. It describes a one to one mapping between search cost and reservation value. This mapping can be shown to be monotone and decreasing. Those properties are fully preserved in our model: the function $\phi$ defined in equation (17) is indeed monotone and decreasing.

In Section B. 3 we will see how the invertibility of $\phi$ is the crucial step in the estimation routine we propose. Unlike the theoretical properties we just described, compared to the standard search model, the computational feasibility of the inversion of $\phi$ is undermined in our setup. This is a consequence of the multi-dimensional integral in equation (17). To overcome this issue we build on some approximation results by Fenton (1960) that together with some standard parametric assumptions will make the $\phi$ inversion fast and feasible during estimation ${ }^{39}$

Model simulation and comparative statics. In this subsection we present some results from model simulations. The broad goal is to give an overview of how the model works and what are some economic patterns it generates.

To be more precise we aim at doing two things. First, we discuss some of the properties of the reservation values as function of search costs $v^{*}=\phi^{-1}(c)$. Second, we consider two scenarios one in which borrowers are relatively more sensitive to periodic payments ( $\alpha_{m}>\alpha_{r}$ ) and the other one in which borrowers are more sensitive to interest rates ( $\alpha_{m}<\alpha_{r}$ ) and compare the implied distributions of realized interest rates and maturities that arise in equilibrium $4^{40}$

Figure 9 plots the relationship between reservation values and search costs that arises by inverting the relationship defined in equation (17). As expected this relationship is monotone, decreasing and convex. The fact that $v^{*}$ is monotonically decreasing in $c$ is intuitive: the higher a borrower search cost the lower the smallest utility value that the borrower is willing to accept. Convexity is a consequence of the option value of search embedded in this type of optimal stopping problem. As $c$ increases, $v^{*}$ becomes linear in $c$ and tends to $\mathbb{E}[\tilde{u}]-c$. This implies

[^20]

Figure 9: Reservation values as function of search costs.
that as search cost increases relatively to the uncertainty in $\tilde{u}$ the option value of waiting and search for an additional lender goes to zero. On the other hand, if search costs are low relative to the uncertainty in $\tilde{u}$ the option value of searching becomes increasingly higher.

Next we turn to the comparison of the two sensitivity scenarios. When consumers respond relatively more to their contract periodic payments ( $\alpha_{m}<\alpha_{r}$ ), Figure 9 shows that, for a given search cost $c$, the corresponding reservation value is lower. This is a direct consequence of the amortizing feature of this type of contracts which, as discussed in Appendix B.5, implies that the distribution of payments is less dispersed than the corresponding distribution of interest rates. In turn when borrowers weigh periodic payments relatively more the distribution of $\tilde{u}$ will also be relatively more compressed and therefore the option value of search will be lower. Overall, borrowers with low $\alpha_{r}$ will search less.

Figure 10 compares the equilibrium distribution of maturities under the two scenarios we are analyzing. We are assuming there are $T=5$ possible maturities from which borrowers can choose. This is to mimic the observed maturities in the data. From figure 1 we can see that $10,15,20,25$ and 30 years are the most common maturities. Going back to figure 10 we can immediately see that when borrowers are more responsive to interest rates low term contract are chosen more frequently. The opposite holds when borrowers care more about their periodic payments. These predictions are in line with the affordability vs. interest cost trade-off embedded in these type of contracts: longer maturities are more attractive to borrowers who would like to smooth their payments as much as possible, while shorter maturities are selected when borrowers want to minimize the total cost of their loan. The predictions in figure (10) also suggest that the observed equilibrium shares of contract terms will be key to inform the price sensitivity parameters of our model.


Figure 10: Loan term equilibrium distribution. Left panel: $\alpha_{r}>\alpha_{m}$. Right panel: $\alpha_{r}<\alpha_{m}$


Figure 11: Equilibrium distribution of realized interest rates. Left panel: PDF. Right panel: CDF.

| Scenario \# | 10 years |  | 15 years |  |  | 20 years |  |  | 25 years |  |  | 30 years |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Mean | Std. | Mean | Std. | Mean | Std. | Mean | Std. | Mean | Std. |  |  |  |  |
| $\left(\alpha_{r}>\alpha_{m}\right)$ | -2.72 | 0.92 | -1.74 | 0.97 | -0.49 | 1.08 | 0.21 | 0.84 | 1.19 | 1.10 |  |  |  |  |
| $\left(\alpha_{r}<\alpha_{m}\right)$ | -2.03 | 0.85 | -1.14 | 1.01 | -0.28 | 0.90 | 0.91 | 0.97 | 1.93 | 1.07 |  |  |  |  |

Table 8: First and second moments of the distribution of realized rates at each maturity. For the same loan term, when $\alpha_{m}>\alpha_{r}$ realized rates are on average higher because search incentives are lower.

Finally, in figure 11 we contrast the equilibrium distributions of interest rates under the two scenarios considered. The unconditional equilibrium distribution is shifted toward higher rates when borrowers are more sensitive to their contract periodic payment. This is the consequence of two complementary forces. First, when $\alpha_{m}>\alpha_{r}$ higher maturities will be chosen more frequently and thus interest rates will be higher. Second, borrowers will have less incentive to search and thus be willing to accept higher interest rates. To separate this second effect from the first one should look at the equilibrium interest rate distribution conditional on a given maturity. Table B.2 shows that this right shift in the equilibrium distribution of interest rates also holds conditional on each of the possible maturities.

Before concluding one final important remark is in order here. In the two scenarios presented each borrower's search cost has been held constant. The reduction in search incentives is purely driven by the increased periodic payments sensitivity. In this model, consumers search less not because it is costly to them but because they might care more about a price dimension (i.e., periodic payments) for which search is less valuable.

## B. 2 Search model identification

In this section we turn the spotlight on the identification of the model we have been describing in the previous section. The goal is to present clear relationships between model parameters and some of the moments we intend to target. To be more precise, the identification exercise we will be proposing works as follows: for a given set of moments we ask how these moments react when we change one of the model parameters while keeping all the others fixed. This will give us an idea on what are the most responsive moments to a given parameter which in turn, looking at the other side of the coin, will tells us what are the data moments that will help pin down that particular model parameter.

Parametric assumptions. To start with, we impose some parametric assumptions and define the model parameters we will be looking at. We assume that the distribution of offered rates at maturity $t$ is given by $r_{t} \sim N\left(\mu_{t}, \sigma_{r}\right)$ for all $t \in\{1, \ldots, T\}$ with $\mu_{t}$ increasing in $t$. The idiosyncratic shocks will instead be distributed as $\varepsilon \sim \operatorname{T1EV}(0,1)$. Finally, we assume the search
costs are $\log$ normal $c \sim \exp \left(N\left(\mu_{c}, \sigma_{c}\right)\right)$. The set of parameters we aim at estimating is given by $\theta=\left(\alpha_{m}, \alpha_{r}, \mu_{c}, \sigma_{c}\right){ }^{411}$

Target moments. For a given value of $\theta$ the model generates a joint distribution of realized interest rates and loan terms $F(r, t)$. Under the above parametric assumptions we should, in principle, be able to write down this likelihood. Nonetheless, the complexity of the problem makes the use of MLE unpractical. Based on the discussion in Section B. 1 we will instead aim at matching the set of moments $m(\theta)=\left(P_{t}(\theta), \mu_{r t}(\theta), v_{r t}(\theta)\right)_{t=1}^{T}$ defined as follows:

- the share of contracts with maturity $t: P_{t}(\theta)$ for all $t \in\{1, \ldots, T\}$,
- the first moment of the distribution of realized interest rates at maturity $t: \mu_{r t}(\theta)$ for all $t \in\{1, \ldots, T\}$,
- the second moment of the distribution of realized interest rates at maturity $t: v_{r t}(\theta)$ for all $t \in\{1, \ldots, T\}$.

Intuitively, the distribution of maturities should inform the price sensitivity parameters $\alpha_{m}$ and $\alpha_{r}$ while the moments of the realized distribution of rates (and/or payments) will tells us about first and second moments of the search cost distribution. In our data we observe the whole joint distribution of interest rate and loan terms so, in practice, there is many other moments that we could match: we are over-identified.

Figures (12) and (13) show how some of the target moments vary as we change borrowers payments sensitivity $\alpha_{m}$ and the location of the search cost distribution $\mu_{c}$ respectively, while keeping all other parameters fixed. In both graphs the first row plots the share of contract at each of the 5 possible maturities $\operatorname{Pr}_{t}(\theta)$ whereas the second row plots the average realized rate (in $\%$ points) at each maturity $\mu_{r t}(\theta)$ for $t \in\{10,15,20,25,30\}$. The shares of contract at each maturities are much more responsive to changes in $\alpha_{m}$ than to changes in $\mu_{c}$ thus suggesting that these set of moments will help pin down $\alpha_{m}$. As expected when we reduce borrowers sensitivity to payments lower maturities are chosen more frequently and longer maturities are less common. Focusing on the second rows, the average realized rate is instead much more responsive to changes in $\mu_{c}$ than to changes in $\alpha_{m}$. As the average search cost increase borrowers will search less and higher rates will be accepted more frequently. Therefore the equilibrium distribution of rates will help inform the search cost parameter $\mu_{c}$.

[^21]

Figure 12: Target moments as a function of $\alpha_{m}$ while keeping the other parameters fixed. Top: shares of contracts at each of the $T=5$ maturities. Bottom: average realized rate at each of the $T=5$ maturities. Other parameters values: $\alpha_{r}=1, \mu_{c}=-1.5, \sigma_{c}=0.5$.


Figure 13: Target moments as a function of $\mu_{c}$ while keeping the other parameters fixed. Top: shares of contracts at each of the $T=5$ maturities. Bottom: average realized rate at each of the $T=5$ maturities. Other parameters values: $\alpha_{r}=1, \alpha_{m}=4, \sigma_{c}=0.5$.

## B. 3 Search model estimation

In this section we do two things. First, we will provide a description of how the model can be estimated. Second, we will discuss the computational feasibility of some of the estimation steps with a particular focus on the inversion of the optimal search policy $\phi$.

Estimation routine. In a nutshell, the goal of our estimation procedure will be to minimize a GMM objective that matches the data moments with the model implied moments. The complexity stems from the fact that within each GMM iteration we need to solve our model, generate the moments we are looking for and finally compute the GMM objective. For a given set of parameter values $\theta$, this procedure requires to nest within each GMM iteration the inversion of the $\phi$ function defined in equation (17). The latter is the computationally most expensive step. The next subsection will be entirely devoted to describe how we can speed up this inversion step during the estimation. The estimation routine we propose is composed by the following steps:
i. Guess parameter values $\theta$.
ii. Draw $c_{i} \sim F_{c}(c ; \theta)$ for $i \in\{1, \ldots, S\}$, where $S$ is a large number of simulated borrowers.
iii. Invert the optimal search policy to obtain borrowers reservation utility $v_{i}^{*}=\phi^{-1}\left(c_{i} ; \theta\right)$ for each $i \in\{1, \ldots, S\}$
iv. Draw borrowers realized utilities $\tilde{u}_{i} \sim F_{\tilde{u}}\left(\tilde{u} \mid \tilde{u}_{i} \geq v_{i}^{*}, \theta\right)$ for each $i \in\{1, \ldots, S\}$.
v. For each $(i, t)$ draw $\delta_{j t} \sim F_{\delta}(\boldsymbol{\delta} \mid t ; \theta)$ and $\varepsilon_{i j t} \sim F_{\varepsilon}$ for all $t \in\{1, \ldots, T\}$ and $j \in\{1, \ldots, J\}$ with $J$ large.
vi. For each $(i, j)$ compute $\tilde{u}_{i j}=\max _{t}\left\{d_{j t}+\varepsilon_{i j t}\right\}$ and pick $j(i)$ such that $\tilde{u}_{i j(i)}$ is the closest to $\tilde{u}_{i}$ in step iv.
vii. For each $i$, let $t_{i}$ be the maturity $t$ such that $\delta_{j(i) t}+\varepsilon_{i j(i) t}=\tilde{u}_{i j(i)}$ and from $\delta_{j(i) t_{i}}$ obtain $r_{i}=r_{j(i) t} .^{42}$
viii. Finally, given $\left(r_{i}, t_{i}\right)_{i=1}^{S}$ compute model-implied moments $m(\theta)$ and obtain GMM objective $(\hat{m}-m(\theta))^{\prime} W^{-1}(\theta)(\hat{m}-m(\theta))$ where $\hat{m}$ is the vector of data moments we want to match.

Among the above steps, the crucial ones are (iii.) and (iv.). Step (iii.) corresponds to the inversion of equation (17) while step (iv.) require us to draw from a non-standard truncated distribution. Next, we discuss the two in details.

[^22]Step (iii.): search policy inversion. Depending on how well we would like to approximate the multi-dimensional integral in equation (17) the inversion of $\phi$, while perfectly feasible, might take from 3-5 seconds to almost 1 minute ${ }^{43}$ Within each GMM iteration this inversion must be performed for each of the $S$ simulated individuals (even though this can be easily parallelized). Moreover, this procedure must be repeated for all the objective function evaluations until convergence. Overall, it might take several hours or even days to run the whole estimation routine.

To speed up computation we will rely on some approximation results first proposed by Fenton (1960). This will allow us to perform the $\phi$ inversion only once outside the estimation. The latter is an extension of what Kim et.al. (2010) use to speed up their estimation exercise. In a nutshell, exploiting the parametric assumptions we made, it is possible to rewrite equation (17) in a standardized way that allows us to perform the inversion outside the estimation for a given grid of search costs $c$ and a variance parameter $\sigma_{\delta}$ we will soon introduce. During estimation, for a given $c$ and $\sigma_{\delta}$ we just interpolate on the pre-defined bivariate grid to obtain the corresponding reservation value.

To start with, we show how to reduce the dimensionality of the problem. First, note that for large maturities and small interest rates we can approximate the periodic payment at maturity $t$ as. 44

$$
\begin{equation*}
m_{t} \approx \frac{1}{k t}+\frac{r_{t}}{2 k} \tag{18}
\end{equation*}
$$

where $k$ is the period frequency ${ }^{45}$ This together with the assumption that $r_{t} \sim N\left(\mu_{r t}, \sigma_{r}\right)$ implies that $\delta_{t} \sim N\left(\mu_{\delta t}, \sigma_{\delta t}\right)$ where $\mu_{\delta t}$ and $\sigma_{\delta t}$ will be function of the price sensitivity parameters $\alpha_{r}$ and $\alpha_{m}$. Now let us define the following variable:

$$
\begin{equation*}
\delta \equiv \log \left(\sum_{t=1}^{T} \exp \left(\delta_{t}\right)\right) \tag{19}
\end{equation*}
$$

and note that with $\varepsilon \sim \operatorname{T1EV}(0,1)$ we have that conditional on $\left(\delta_{1}, \ldots, \delta_{T}\right), \tilde{u} \sim T 1 E V(\delta)$, with $\delta$ as in (19) above. Therefore, the distribution of $\tilde{u}$ depends on $\left(\delta_{1}, \ldots, \delta_{T}\right)$ only through $\delta$. This reduces the dimensionality of the integral in equation from $T+1$ dimensions to 2

[^23]dimensions only. Thus, we can rewrite (17) as follows:
\[

$$
\begin{align*}
c=\phi\left(v^{*}\right) & =\mathbb{E}_{\delta}\left[\int_{v^{*}}^{\infty}\left(\tilde{u}-v^{*}\right) f_{\tilde{u}}(\tilde{u} \mid \boldsymbol{\delta}) d \tilde{u}\right] \\
& =\int_{-\infty}^{\infty} \int_{v^{*}}^{\infty}\left(\tilde{u}-v^{*}\right) f_{\tilde{u}}(\tilde{u} \mid \boldsymbol{\delta}) f_{\delta}(\boldsymbol{\delta}) d \tilde{u} d \boldsymbol{\delta} \\
& =\int_{-\infty}^{\infty}\left(1-F_{\tilde{u}}\left(v^{*} \mid \boldsymbol{\delta}\right)\right)\left(\mathbb{E}\left[\tilde{u} \mid \tilde{u} \geq v^{*}, \boldsymbol{\delta}\right]-v^{*}\right) f(\boldsymbol{\delta}) d \boldsymbol{\delta} \tag{20}
\end{align*}
$$
\]

where $f_{\tilde{u}}(\tilde{u} \mid \delta)$ is the pdf of a T1EV with location parameter $\delta$. Compared to (17) the inversion of $(20)$ is way more feasible.

What we did so far makes the inversion of the function $\phi$ perfectly feasible. In principle, for given search cost $c$ nothing more is needed to numerically compute the associated reservation value $v^{*}$. Yet, as we discussed, speeding up the inversion of within our estimation can greatly improve computational efficiency. Next, we discuss how we make this inversion feasible even when the estimation runs.

Recall that $\delta_{t} \sim N\left(\mu_{\delta t}, \sigma_{\delta t}\right)$ and that $\delta=\log \left(\sum_{t=1}^{T} \exp \left(\delta_{t}\right)\right)$. Fenton (1960) suggests a simple and effective way to approximate the sum of $\log$ normal distributions with a log-normal whose first and second moments are directly derived from the ones of the underlying lognormals. ${ }^{46}$ Thus, we use a single normal distribution to approximate the distribution of $\delta \approx$ $N\left(\mu_{\delta}, \sigma_{\delta}\right)$ so that $\delta=\mu_{\delta}+\sigma_{\delta} \delta_{z}$ with $\delta_{z} \sim N(0,1) \cdot{ }^{[47}$ Next, define the following variable

$$
\tilde{z} \equiv \tilde{u}-\mu_{\delta}
$$

and rearrange 20 as follows:

$$
\begin{equation*}
c=\int_{-\infty}^{\infty}\left(1-F_{\tilde{z}}\left(z^{*} \mid \sigma_{\delta} \delta_{z}\right)\right)\left(\mathbb{E}\left[\tilde{z} \mid \tilde{z} \geq z^{*}, \sigma_{\delta} \delta_{z}\right]-z^{*}\right) f\left(\delta_{z}\right) d \delta_{z}, \tag{21}
\end{equation*}
$$

where $F_{\tilde{z}}$ is the cdf of a T1EV with location parameter $\sigma_{\delta} \delta_{z}$ and $f\left(\delta_{z}\right)$ is the standard normal pdf.

Looking at (21) we note that the inversion will only depend on two parameters: search cost $c$ and the standard deviation $\sigma_{\delta}$. Thus, following the idea in Kim et.al (2010), we point out that for a given, arbitrarily fine, 2-dimensional grid of $\left(c, \sigma_{\delta}\right)$ values we can obtain a standardized reservation value $z^{*}\left(c, \sigma_{\delta}\right)$ by inverting (21). Thus, before estimation we can perform this inversion for all the combinations of $\left(c, \sigma_{\delta}\right)$ in our pre-determined grid and store the corresponding value $z^{*}\left(c, \sigma_{\delta}\right)$. Armed with a table of $\left(c, \sigma_{\delta}\right)$ and $z^{*}\left(c, \sigma_{\delta}\right)$, during estimation we can for $c$ and the current $\sigma_{\delta}(\theta)$ look up $z^{*}\left(c, \sigma_{\delta}\right)$ from the table using an interpolation step in place of step (iii.).

[^24]

Figure 14: Reservation value $z^{*}$ as function of $c$ and $\sigma_{\delta}$ as defined in equation (21).
The surface plotted in figure (14) comes from a typical table. It shows the relationship between $\left(c, \sigma_{\delta}\right)$ and the corresponding $z^{*}\left(c, \sigma_{\delta}\right)$. For given $\sigma_{\delta}$, as $c$ increases, $z^{*}$ is monotonically decreasing and convex exactly as in figure 9 . For given $c$, as $\sigma_{\delta}$ increases $z^{*}$ is monotonically increasing. The latter is intuitive: the higher the dispersion/uncertainty of the distribution along which search is performed, the higher the option value of searching and in turn the higher the reservation value. Finally, going back to estimation, given the current guess $\theta$, from $z^{*}\left(c, \sigma_{\delta}(\theta)\right)$ we can back out the true reservation value as $u^{*}=z^{*}+\mu_{\delta}(\theta)$ for each of the simulated individuals.

Step (iv.): drawing from $F_{\tilde{u}}$. The distribution of the maximum utilities $F_{\tilde{u}}$ is non-standard:

$$
\begin{equation*}
F_{\tilde{u}}(x)=\int_{-\infty}^{\infty} F_{\tilde{u}}(x \mid \boldsymbol{\delta}) f_{\delta}(\boldsymbol{\delta}) d \boldsymbol{\delta} \tag{22}
\end{equation*}
$$

where $\delta$ is as in (19) and $F_{\tilde{u}}(\tilde{u} \mid \boldsymbol{\delta})$ is $\operatorname{T1EV}(\delta)$. In order to draw from this distribution in an efficient way we again rely on Fenton (1960) and approximate the distribution of $\delta$ with a normal $N\left(\mu_{\delta}, \sigma_{\delta}\right)$. In this way, drawing from (22) becomes way more practical and efficient.

First, outside the estimation, we draw a sample of dimension $S$ from an uniform distribution $U_{i} \sim U[0,1]$ with $i \in\{1, \ldots, S\}$. Second, given $\mu_{\delta}(\theta)$ and $\sigma_{\delta}(\theta)$, we transform those draws as follows: $\xi_{i}=F_{\tilde{u}}\left(v_{i}^{*}\right)+\left(1-F_{\tilde{u}}\left(v_{i}^{*}\right)\right) U_{i}$ for $i \in\{1, \ldots, S\}$ and finally, inverting 22, we obtain
$\tilde{u}_{i}=F_{\tilde{u}}^{-1}\left(\xi_{i}\right)$ which will be distributed as $F_{\tilde{u}}\left(\tilde{u} \mid \tilde{u} \geq v_{i}^{*}\right)$ for $i \in\{1, \ldots, S\}{ }^{48}$

## B. 4 Search model microfoundation

In section (B.1), following the literature in demand estimation, we assumed borrowers preferences for loan $j$ with rate $r$ and maturity $\tau$ are described by the following random utility model:

$$
\begin{equation*}
u_{i j \tau}=x_{j t} \beta-\alpha_{m} m_{j \tau}-\alpha_{r} r_{j \tau}+\varepsilon_{i j \tau} \tag{23}
\end{equation*}
$$

with $\varepsilon_{i j \tau} \sim \operatorname{T1EV}(0,1)$. After visiting lender $j$, borrower $i$ will select the loan term $t$ that maximizes 23) for a discrete set of maturities $T$ which makes the use of random utility models quite attractive. In this section we first discuss the microfoundation of this type of random utility models for regular products and then discuss it for more complex products such as loans and other financial contracts.

Regular products. For the case of non-financial products the microfoundation of random utility models has been discussed extensively in the literature ${ }^{49}$ Suppose, consumer $i$ 's utility is defined over $J$ products and an outside good 0 . Moreover, assume $i$ buys at most one unit of $j$ i.e., $q_{j} \in\{0,1\}$ for all $j \in\{1, \ldots, J\}$. Consumer $i$ 's problem is given by:

$$
\begin{gathered}
\max _{\left(q_{j}\right)_{j=1}^{J} \in\{0,1\}^{J}, q_{0} \geq 0} V\left(U\left(q_{1}, \ldots, q_{J}\right), q_{0}\right) \\
\text { s.t. } q \cdot p \leq y
\end{gathered}
$$

or equivalently, under monotonicity of preferences,

$$
\max _{\left(q_{j}\right)_{j=1} \in\{0,1\}^{J}} V\left(U\left(q_{1}, \ldots, q_{J}\right), y-\sum_{j=1}^{J} q_{j} p_{j}\right) .
$$

Next assuming the $J$ products are perfect substitutes $U\left(q_{1}, \ldots, q_{J}\right)=\sum_{j=1}^{J} \phi_{j} q_{j}$, consumer $i$ 's problem boils down to a discrete choice problem among the following choice-specific indirect utilities:

$$
\begin{equation*}
V\left(\phi_{j}, y-p_{j}\right) \quad \text { all } j \in\{1, \ldots, J\} \quad \text { and } \quad V(0, y) . \tag{24}
\end{equation*}
$$

From (24) to obtain the typical indirect utilities as the one in (23) the econometrician needs to assume quasi-linearity of preferences in the outside good and add an unobserved error compo-

[^25]nent. Consumer $i$ 's indirect utility from alternative $j$ is thus given by.$\sqrt{50}$
\[

$$
\begin{equation*}
u_{i j}=V\left(\phi_{j}\right)-\alpha p_{j}+\varepsilon_{i j} \quad \text { all } j \in\{1, \ldots, J\} . \tag{25}
\end{equation*}
$$

\]

Financial products. The microfoundation of random utility models for products like mortgages, credit cards, personal loans etc. is inevitably more complex. The first reason is that the purchase of these products usually requires consumers to make dynamic and forward looking considerations that interacts with other consumption/savings decisions. This is because these contracts typically generate a cash-flow that requires borrower to make periodic payments to their lender. The second reason is that there is not a clear one dimensional price for these products. The interest rate is a measure of the overall cost of the loan but it is uninformative about the impact of loan payments on consumers budget. These periodic payments are another important price dimension that influences consumer choices. Overall, the perceived loan price can be quite subjective: some individuals might weight more the interest rate while others might weight more periodic payments.

This subsection discusses how to characterize consumers indirect utility as a function of loan rate and loan maturity from the perspective of a consumption/saving problem in which housing is the only asset.

Consumption-saving problem. Consider a consumer whose utility is defined over an housing good $h_{0}$ and a consumption stream $\left(c_{t}\right)_{t=0}^{\infty}$. To finance its housing good purchase at time 0 she needs to take up a loan with maturity $\tau$ at rate $r_{\tau}$. The dynamic problem is defined as follows:

$$
\begin{aligned}
V\left(\tau, r_{\tau}\right) \equiv \max _{h_{0}, c_{t}} & \sum_{t=0}^{\infty} \beta^{t} u\left(h_{0}, c_{t}\right) \\
& \text { s.t. } m_{0 \tau} q_{0} h_{0}+c_{t} \leq y_{t}+q_{t} h_{0} \tilde{h}_{t \tau} \quad \text { for } t<\tau \\
& c_{t} \leq y_{t}+q_{t} h_{0} \quad \text { for } t \geq \tau
\end{aligned}
$$

where $m_{0 \tau}$ is the per $\$$ periodic payment, $\tilde{h}_{t \tau}$ is the per \$ house equity built up to time $t$ and $q_{t}$ is the house price. We assume $y_{0}<q_{0} h_{0}$ so that the only way housing can be positive $h_{0}>0$ together with consumption $c_{0}>0$ is by taking up a mortgage. Moreover, we assume that the loan cannot be repaid before maturity.

Next, the goal is to characterize how our consumer indirect utility $V\left(\tau, r_{\tau}\right)$ changes with $r_{\tau}$ and $\tau$ respectively. Let $\lambda_{t}$ be the Lagrange multiplier, applying the envelope theorem one obtains:

[^26]\[

$$
\begin{align*}
& \frac{d V\left(\tau, r_{\tau}\right)}{d r_{\tau}}=h_{0} \sum_{t=0}^{\tau-1} \beta^{t} \lambda_{t}\left(q_{t} \frac{d \tilde{h}_{t \tau}}{d r_{\tau}}-q_{0} \frac{d m_{0 \tau}}{d r_{\tau}}\right)  \tag{26}\\
& \frac{d V\left(\tau, r_{\tau}\right)}{d \tau}=h_{0} \frac{d}{d \tau}\left[\sum_{\tau=0}^{\tau-1} \beta^{t} \lambda_{t}\left(q_{t} \tilde{h}_{t \tau}-q_{0} m_{0 \tau}\right)\right] . \tag{27}
\end{align*}
$$
\]

Not surprisingly equation (26) is negative. This is because the periodic payments $m_{0 \tau}$ increases in the interest rate while the amount of house equity built at time $\tilde{h}_{t \tau}$ decreases at each $t$ as higher rates reduce the pace of equity accumulation. As expected, consumer indirect utility decreases in the rate paid. Turning to the loan term, the sign of equation (27) is ambiguous. Higher $\tau$ is beneficial because $m_{0 \tau}$ decreases as periodic payments are lower but the house equity built at each $t, \tilde{h}_{t \tau}$ is lower as interest is repaid over a longer horizon.

To sum up, from this simple example we can draw at least three conclusions. First, while consumers always prefer lower rates, the mechanism through which lower $r_{\tau}$ reduces $V$ is made of multiple components. On one hand a per-period gain from lower payments and higher equity, on the other a forward looking gain that through $\beta$ and $\lambda_{t}$ cumulates the per-period gain. Second, whether or not lower maturities are beneficial is ambiguous. Higher maturities generate a per-period gain through lower $m_{0 \tau}$ which is contrasted by a per-period loss from lower equity ${ }^{51}$ This ambiguous effect translates into a forward looking effect through $\beta$ and $\lambda_{t}$. Finally, it is clear that with only market data at the time of loan issuance it is unfeasible to separately quantify all these effects. Yet, the example underlies the importance of endogenizing the maturity choice and including the periodic payments a component of consumers indirect utility.

Quasi-linear preferences. In this subsection we further simplify the above consumption saving problem in order to gain intuition about consumers indirect utility over loan rates and maturities. The assumptions we make will allow us to focus on two relevant channels through which a consumer might prefer lower interest rates: the interest channel and the budget channel. The relative importance of the two channels will be key to pin down the optimal maturity of the contract. Consider the previous consumption-saving problem and assume the following:

- quasi-linearity: $u\left(h_{0}, c_{t}\right)=u\left(h_{0}\right)+c_{t}$,
- consumer cannot borrow against partial house-equity e.g., $\tilde{h}_{t \tau} \equiv 0$.

Under the two above assumptions $\lambda_{t}=1$ for all $t$ and consumer indirect utility reads:

$$
\begin{equation*}
V\left(\tau, r_{\tau}\right)=\frac{u\left(h_{0}\right)}{1-\beta}-q_{0} h_{0} \sum_{t=0}^{\tau-1} \beta^{t} m_{0 \tau} \tag{28}
\end{equation*}
$$

[^27]Starting from (28) we can perform again the previous comparative static:

$$
\begin{align*}
& \frac{d V\left(\tau, r_{\tau}\right)}{d r_{\tau}}=-q_{0} h_{0} \sum_{t=0}^{\tau-1} \beta^{t} \frac{d m_{0 \tau}}{d r_{\tau}}<0  \tag{29}\\
& \frac{d V\left(\tau, r_{\tau}\right)}{d \tau}=-q_{0} h_{0} \frac{d}{d \tau} \sum_{t=0}^{\tau-1} \beta^{t} m_{0 \tau} \tag{30}
\end{align*}
$$

The optimal maturity choice is given by $\tau^{*}(\beta)$ such that (30) equals 0 . In addition, it easy to show that $\tau^{*}(\beta)$ is decreasing in $\beta$.

In this example everything is determined by the discount rate $\beta$. When $\beta \approx 0$ our consumer will pick the highest possible $\tau$ as 30 reduces to $-q_{0} h_{0} \frac{d m_{0 \tau}}{d \tau}>0$. On the contrary when $\beta \approx 1$ equation (30) is proportional to $-\tau \frac{d r_{\tau}}{d \tau}-r_{\tau}<0$ so that our consumer will choose the lowest feasible maturity.

The interpretation of these results is straightforward. When consumers are myopic (e.g., $\beta \approx 0$ ) the benefit from a lower rate only enters $V$ through the first term $m_{0 \tau}$ of the discounted flow of payments. Consumer increase in indirect utility comes from a relaxation of their budget constraint ${ }_{52}^{52}$ this what we call the budget channel. On the other hand, when consumer are forward looking (e.g., $\beta \approx 1$ ) the benefit from a lower rate enters the indirect utility proportionally to $\tau r_{\tau}$ (i.e., the total interest cost): this is what we call the interest channel. Which of the two channels dominates depends on $\beta$ and in turn determines what is going to be the optimal maturity choice.

To conclude, this example highlights two things. First, the price dimension that consumer weigh more (e.g., payments vs. interest rate) is what determines the choice of the loan term. Second, how consumers evaluate pairs $\left(\tau, r_{\tau}\right)$ can be thought as some sort of average between per-period gains and long term gains. Overall, we believe that while not perfectly microfounded our random indirect utility in (15) should capture this trade-off and at the same time enable us to have a model that can be estimated with market data only.

## B. 5 Appendix: Price dispersion comparison

In this Appendix we show that for a given distribution of interest rates (at a given maturity) that borrowers face in the market the corresponding market distribution of periodic payments is relative less dispersed. The amortizing feature of these loans generates a wedge between the dispersion of these two distribution.

Let $C$ be the amount borrowed, $k$ the interest compounding frequency in each period (e.g., for monthly payments $k=12$ in one period), $r$ the interest rate and $\tau$ the contract term. The

[^28]periodic payment can be computed as:
\[

$$
\begin{align*}
m(C, r, \tau)=C \times m(r, \tau) & =C \times \frac{r / k}{1-(1+r / k)^{-\tau k}}  \tag{31}\\
& \approx C \times\left(\frac{1}{\tau k}+\frac{t k+1}{2 t k} \frac{r}{k}\right)  \tag{32}\\
& \approx C \times\left(\frac{1}{\tau k}+\frac{r}{2 k}\right) \tag{33}
\end{align*}
$$
\]

where the second equality applies the mortgage french amortization formula, the first approximation applies a first order Taylor expansion around $(r / k=0)$ and the third approximation is for $\frac{t k+1}{2 t k} \approx 1 / 2{ }^{53}$

Given the above it is immediate to see that for given $\tau$ the periodic payment per $\$$ borrowed is less dispersed that the interest rate ${ }^{54}$

$$
\begin{equation*}
\operatorname{Var}(m(r, \tau))=\frac{\operatorname{Var}(r)}{4 k^{2}}<\operatorname{Var}(r) \tag{34}
\end{equation*}
$$

Condition (34) suggests that for each $\$$ borrowed on a loan at a given maturity, expected savings in term of interest are higher than expected savings in term of payments when shopping around across different lenders.

Instead of looking at payments per $\$$ borrowed, one can also show that payments in $\$$ are relative less dispersed than interest rates. To see this we compute the coefficient of variation of $m(C, r, \tau)$ and compare it with the coefficient of variation of $r$ :

$$
\begin{aligned}
\frac{m(C, r, \tau)}{\mathbb{E}[m(C, r, \tau)]} & =\frac{m(r, \tau)}{\mathbb{E}[m(r, \tau)]} \\
& =\frac{1}{\tau k \mathbb{E}[m(r, \tau)]}+\left(\frac{\mathbb{E}[r]}{2 k \mathbb{E}[m(r, \tau)]}\right) \frac{r}{\mathbb{E}[r]} \\
& =\frac{1}{\tau k \mathbb{E}[m(r, \tau)]}+\left(\frac{\mathbb{E}[r]}{2 \tau^{-1}+\mathbb{E}[r]}\right) \frac{r}{\mathbb{E}[r]}
\end{aligned}
$$

from which

$$
\begin{equation*}
\frac{s d(m(C, r, \tau))}{\mathbb{E}[m(C, r, \tau)]}<\frac{s d(r)}{\mathbb{E}[r]} . \tag{35}
\end{equation*}
$$

The interpretation is the same as before: in relative terms the distribution of payments is less dispersed than the distribution of rates.

Implication for search incentives. We now propose an heuristic argument to show how (34) or (35) shape search incentives.

[^29]To start with suppose there are two borrowers 1 and 2 with the same search cost $c$ and whose lowest rate searched so far is $r{ }^{55}$ The only difference is that 1 searches along the distribution of rates while 2 searches along the distribution of payments per $\$$ borrowed. Then the expected savings from an additional search for 1 are higher than for 2 :

$$
\begin{align*}
M B S_{1}(r) & =\int_{\underline{\underline{r}}}^{r}(r-\tilde{r}) d F(\tilde{r})  \tag{36}\\
& =2 k \int_{\underline{m}}^{m}(m-\tilde{m}) d F(\tilde{m})  \tag{37}\\
& >\int_{\underline{m}}^{m}(m-\tilde{m}) d F(\tilde{m})=M B S_{2}(r) \tag{38}
\end{align*}
$$

so that, all else equal, 1 will have more incentive to search compared to 2 .

[^30]
[^0]:    *The views expressed are those of the authors and do not necessarily reflect those of the ECB or the Eurosystem. The paper has been started while Alessandro Ferrari was employed at the Bank of Italy which we acknowledge for data from Credit register and Mutuionline. We are thankful to Carlo Altavilla, Stéphane Bonhomme, Vincenzo Cuciniello, Hazen Eckert, Lars Hansen, Eyo Herstad, Ali Hortaçsu, Valentina Michelangeli, Scott Nelson, Stefano Neri, Alessandro Secchi, Federico M. Signoretti, Andrea Tiseno for helpful comments and conversations.
    ${ }^{\dagger}$ European Central Bank, DG Monetary Policy, Monetary Analysis Division, alessandro.ferrari @ecb.europa.eu
    ${ }^{*}$ Department of Economics, University of Chicago, mloseto@uchicago.edu. Financial support from the Bank of Italy Bonaldo Stringher scholarship is gratefully acknowledged.

[^1]:    ${ }^{1}$ In the US, almost all mortgages issued have maturity of either 15 or 30 years, with the latter being the most common contract term.
    ${ }^{2}$ Another way in which borrowers can improve their loan conditions is by shopping around across lenders. In appendix $B$ we show that there is indeed substantial dispersion in both realized and offered interest rates suggesting that searching across lenders might be worthwhile. The dispersion in rates can be rationalized by introducing search frictions. In appendix B we develop a structural model in which borrowers search across lenders that offers different combinations of interest rates and maturities.
    ${ }^{3}$ Through the MutuiOnline.it platform potential borrowers can search, apply for and obtain a mortgage.

[^2]:    ${ }^{4}$ We are not the first to exploit maturity elasticities to test for the presence of credit constraints. For auto-loans Attanasio et al. (2008) and Argyle et al. (2020) also find that on average consumers are more sensitive to their contract maturity than to interest rate charges consistent with the presence of binding credit constraints.

[^3]:    ${ }^{5}$ The literature that estimates maturity elasticities of household debt has focused on auto-loans (Juster and Shay (1964), Attanasio et al. (2008), Argyle et al. (2020)) and student loans (Bachas (2019)). One exception is the working paper by Maiser and Villanueva (2011) where they use survey panel data to study how the consumption responds to initial mortgage conditions including maturity. Our paper differs in several dimensions. First, we use administrative data on the universe of mortgage contracts issued by each lender in each province which allows us to take into account any unobserved product quality across different lenders and unobserved shocks to different local markets. Second, we focus on the response of loan demand which can be measured with much more precision than consumption expenditure. Third, we exploit high frequency variation in posted lenders' price schedules as a source of identification instead of averages of borrowers' self-reported interest rates.

[^4]:    ${ }^{6}$ See for instance the recent discussion in Guiso et al. (2021) Online Appendix A1.
    ${ }^{7}$ Additionally, most of Italian mortgages are full-recourse which lowers borrowers' option value of defaulting.
    ${ }^{8}$ In appendix B , after introducing a structural model that includes search frictions and discrete maturity choice, we microfound the model in from a simple consumption-savings problem with quasi-linear preferences and housing asset.
    ${ }^{9}$ The hand-to-mouth assumption is not unreasonable for italian mortgage holders. Evidence from the Survey on Italian Household Income and Wealth (SHIW) suggests that less than $10 \%$ of borrowers hold a positive amount of financial assets and that on average $80 \%$ of net disposable income is consumed.

[^5]:    ${ }^{10}$ Our model combines the long-term debt consumption-saving model discussed in Argyle et al. (2020) and the loan repayment model developed in Bachas (2019)
    ${ }^{11}$ Contract length should always be positive $s \geq 0$ yet this constraint will never bind if $b_{0}^{*}>0$ as the monthly payment required would be infinite.
    ${ }^{12}$ The continuous-time limit of the periodic payment function is given by $m(r, \tau) \equiv \frac{r}{1-\rho^{-r \tau}}$.
    ${ }^{13}$ The economics behind the optimal choice of maturity is analogous to the one Bachas (2019) develops for the repayment of student loans.

[^6]:    ${ }^{14}$ For some particular combination of parameters there can be borrowers that are slightly constrained $\mu>0$ and yet do not find it worth it to increase maturity up to the maximum available. These subgroup of constrained borrowers will select an interior maturity $s<\tau$ and will be unresponsive to marginal changes in the maximum available maturity $\tau$ which is the focus of our analysis.
    ${ }^{15}$ In Italy banks typically require monthly payment not to exceed $\phi=1 / 3$ of the household monthly income.

[^7]:    ${ }^{16}$ See Appendix A for a detailed description on how we reconstruct the maturity of each contract.
    ${ }^{17}$ Most but not all the Italian lenders post offers on this online platform. Yet all the largest banks do which account for more or less $70 \%$ of mortgage origination. See Carella and Michelangeli (2021) and Carella et al. (2020) for a thorough discussion of the MutuiOnline.it dataset.

[^8]:    ${ }^{18}$ The APR includes the interest rate plus any other lump-sum fee or upfront expense charged on the borrower. The net interest rate (together with the contract duration) is what determines the monthly installment.
    ${ }^{19}$ Differently from the US, in Italy there is no official credit scoring system. LTV is the only measure lenders can use to assess borrowers credit-worthiness together with information on past delinquencies which is available from the Credit Register upon request.
    ${ }^{20}$ In practice the share of contracts that are finalized online through MutuiOnline.it is less than $10 \%$ of the market (Michelangeli et al. (2020))

[^9]:    ${ }^{21}$ Carella et al. (2020) and Michelangeli et al. (2020) provide a detailed discussion on the institutional settings of the online italian mortgage market.

[^10]:    ${ }^{22}$ Here $L=\{50,60,80,85\}$ is the set of LTV that contribute at defining our borrower profile. Similarly, $B$ is the set of possible combination of borrowers' characteristics described in section 4.2

[^11]:    ${ }^{23}$ Italian banks heavily rely on approval screening policies yet, conditional on acceptance, on the intensive margin risk is not priced and only recently the LTV started to slightly matter. See discussion in Guiso et al. (2021).
    ${ }^{24}$ The share of mortgages finalized through the intermediation of MutuiOnline.it is quite small— in 2015 it was about 6\% (Michelangeli et al.(2020)). Moreover, evidence from the SHIW suggests that around 60\% of borrowers took up their mortgage at their usual bank without considering other offers. Overall, it is very likely that online demand is quite different from the one in the main mortgage market e.g., mostly composed by more sophisticated and educated borrowers.
    ${ }^{25}$ This way of reasoning is quite common in the empirical IO literature where prices in other market are used a proxy for unobserved marginal costs as in Nevo (2001).

[^12]:    ${ }^{26}$ Mortgages are typically priced as a spread over some benchmark market rate which reflects banks' opportunity cost of mortgage lending. ARMs are typically priced over the 3 month Euribor whereas FRMs at some given maturity $\tau$ are priced over the corresponding interest rate swap with maturity $\tau$.

[^13]:    ${ }^{27}$ The analysis in DeFusco and Paciorek (2017) covers the US mortgage market for the years in between 1997 and 2007. In that period the average APR for FRM was about 6.5\%.

[^14]:    ${ }^{28}$ For the US mortgage market Dhillon et al. (1990) show empirically that wealthier borrowers are more likely to choose a 15 years FRM than a 30 years one.

[^15]:    ${ }^{29}|\{m: q(m)=q\}|$ is the number of months of quarter $q$ in which we observe contract $i$ 's outstanding balance. This number is typically 3 except for the quarter in which the contract has been originated $q(o)$.

[^16]:    ${ }^{30}$ Data on offered rates come from an experiment run on an online platform (MutuiOnline) on which consumers can search, compare and take up mortgages. We refer to Carella and Michelangeli (2021) for a more detailed description of this dataset.
    ${ }^{31}$ See for example Agarwal et.al. (2020), Allen et.al. (2014), (2019) and Bhutta et.al (2021).
    ${ }^{32}$ For the case of non financial products an example of this identification strategy can be found in Hong and Shum (2006).

[^17]:    ${ }^{33}$ See Chari and Jagannathan (1989) and Stanton and Wallace (1998) for a discussion on the trade-off between interest rate and points.
    ${ }^{34} \mathrm{https}: / / \mathrm{www} . b a n k o f a m e r i c a . c o m / a u t o-l o a n s / h o w-c a r-l o a n s-w o r k / ~$

[^18]:    ${ }^{35}$ The argument here is heuristic. To understand search incentives we need to assume how rates and payments enter borrowers preferences. Suppose borrowers cost for a loan with rate $r$ and maturity $\tau$ is the discounted sum of future payments: $u\left(r_{\tau}, \tau\right) \equiv-\sum_{t=0}^{\tau} \beta^{t} m\left(r_{\tau}, \tau\right)$ where $m$ is the periodic payment. Then consumers with $\beta \approx 0$ only search to minimize $m\left(r_{\tau}, \tau\right)$ while people with $\beta \approx 1$ search to minimize $r_{\tau}$. Overall, it is the distribution of $\tilde{u} \equiv \max _{\tau}\left\{u\left(r_{\tau}, \tau\right)\right\}$ that will determine borrowers marginal benefit of an additional search. For a more detailed discussion refer to subsections B. 1 and B. 4

[^19]:    ${ }^{36}$ Here we are relying on the IO approach to span consumers utility on a set of product characteristics. Subsection B. 4 of this appendix discusses more about how to microfound borrowers indirect utility over maturities and rates from the lens of a dynamic consumption saving problem.
    ${ }^{37}$ Note that the product that lenders sell is homogeneous except for the price i.e, the yield curves that lenders posts; by product characteristics we refer to contract terms like the amount borrowed or the type of interest rate (fixed vs. floating) that borrowers can freely customize independently of the lender. In that sense, those characteristics do not differentiate lenders products like we usually think when estimating demand for differentiated products. In this model we only allow borrowers to customize their maturity or equivalently, we assume that the observed choices of other contract terms would have been the same if our borrower were to sign its contract with any other lender.
    ${ }^{38}$ Defining $\alpha_{r}$ as the sensitivity to the interest rate is not entirely precise as it is not possible to keep $m_{j t}$ fixed as we change $r_{j t}$ changes (unless of course we change maturity accordingly to keep $m_{j t}$ constant). Exploiting the approximation derived in equation 18 the sensitivity of $u_{i j t}$ to $r_{j t}$ is given by $\left(\frac{\alpha_{m}}{2 k}+\alpha_{r}\right)$ whereas $\alpha_{r}$ is the change in $u_{i j t}$ for an unit change in $r_{j t}$ compensated by a change in $t$ that keeps $m_{j t}$ fixed.

[^20]:    ${ }^{39}$ Marone and Sabety (2021) also rely on this approximation to estimate their structural model for the choice of financial coverage levels in health insurance market.
    ${ }^{40}$ To be exactly precise, borrowers are more sensitive to payments than to interest rates if and only if $\alpha_{m}>$ $\frac{2 k}{2 k-1} \alpha_{r}$.

[^21]:    ${ }^{41}$ At the moment we are taking $\left(\mu_{t}\right)_{t=1}^{T}$ and $\sigma_{r}$ as known. In practice we aim at using the data on posted rates to pin those down.

[^22]:    ${ }^{42}$ Recall that $\delta_{j t}$ is a linear function of $r_{j t}$.

[^23]:    ${ }^{43}$ It requires around 3 seconds when the number of approximating draws is around 2000-3000 while around 40-50 seconds when the number of approximation draws is around 10,000-20,000.
    ${ }^{44}$ Benetton (2020) uses a similar approximation. Also note that equation 18 already highlights how $m_{t}$ will be less dispersed than $r_{t}$ conditional on $t$, for detail we refer to also Appendix B. 5
    ${ }^{45}$ For instance $k=12$ for the case monthly payments.

[^24]:    ${ }^{46}$ See Fenton (1960) for details. This approximation has been widely used engineering, telecomunications and finance and it is also known as Fenton-Wilkinson approximation. See also Schwartz and Yeh (1982), Ho (1995), Cobb et.al. (2012) and for a recent paper in health economics Marone and Sabety (2021).
    ${ }^{47}$ It would also be feasible to use a flexible mixture of normal yet by virtue of the Fenton-Wilkinson approximation a single normal distribution should work particularly well.

[^25]:    ${ }^{48}$ Here we can also perform the inversion outside the estimation for a given gird of $\sigma_{\delta}$ values.
    ${ }^{49}$ See for instance, Dubé, JP. (2019)'s "Microeconometric Models of Consumer Demand" in the Handbook of the Economics of Marketing.

[^26]:    ${ }^{50}$ The term $V\left(\phi_{j}\right)$ is spanned by the observable product characteristics $x_{j} \beta$. The errors $\varepsilon$ has been interpreted in many different ways: unobserved product characteristics, taste shocks, measurement or specification errors. The recent literature in rational inattention (e.g., Matejka and McKay, 2015) rationalizes this random utility component consumer's product uncertainty and the costs of endogenously reducing uncertainty through search.

[^27]:    ${ }^{51}$ Dhillon (1990) finds evidence that equity motives might push borrower towards lower maturities.

[^28]:    ${ }^{52}$ Recall that the marginal utility of income is constant $\lambda_{t}=1$ for all $t$.

[^29]:    ${ }^{53}$ The first approximation works quite well as $r / k$ is in practice very small. The second approximation is not needed for the results. It is just to make the expression better looking. Nonetheless, this approximation is also quite reliable as typically $k=12$ and $t=10$ at its lowest so $\frac{t k+1}{t k}$ is basically 0.5
    ${ }^{54}$ Recall that here different lenders charge different prices that is why there is a distribution of interest rates.

[^30]:    ${ }^{55}$ The maturity is fixed here.

